TRANSPORTATION NETWORK STABILITY: A CASE STUDY OF CITY TRANSIT

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Motivation

- To study the effects that defunct or removed nodes have on the properties of public transport networks

- To present criteria, that allow to \textit{a priori} quantify the attack stability of real world correlated networks

- Analysis of public transport (PT) networks of various means of transport
cf. Boston subway \textit{(Marichiori, Latora’00-’02)}, Vienna U-Bahn \textit{(Seaton, Hackett’04)}

- Study of PT networks on a larger database
cf. 22 cities in Poland, Warsaw: \( N = 1530 \) \textit{(Sienkiewicz, Hołyst’05)}, Berlin, Düsseldorf, Paris \textit{(von Ferber et al.’05)}
Network interpretation

Public transit map

(Bipartite) B-space

L-space

P-space

C-space
### Observables, $L$-space

<table>
<thead>
<tr>
<th>City</th>
<th>$\langle k \rangle$</th>
<th>$\ell_{\text{max}}$</th>
<th>$\langle \ell \rangle$</th>
<th>$C$</th>
<th>$\kappa^{(z)}$</th>
<th>$\kappa^{(k)}$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berlin</td>
<td>2.58</td>
<td>68</td>
<td>18.5</td>
<td>52.8</td>
<td>1.96</td>
<td>3.16</td>
<td>(4.30)</td>
</tr>
<tr>
<td>Dallas</td>
<td>2.18</td>
<td>156</td>
<td>52.0</td>
<td>55.0</td>
<td>1.28</td>
<td>2.35</td>
<td>5.49</td>
</tr>
<tr>
<td>Düsseldorf</td>
<td>2.57</td>
<td>48</td>
<td>12.5</td>
<td>24.4</td>
<td>1.96</td>
<td>3.16</td>
<td>3.76</td>
</tr>
<tr>
<td>Hamburg</td>
<td>2.65</td>
<td>156</td>
<td>39.7</td>
<td>254.7</td>
<td>1.85</td>
<td>3.26</td>
<td>(4.74)</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>3.59</td>
<td>60</td>
<td>11.0</td>
<td>60.3</td>
<td>3.24</td>
<td>5.34</td>
<td>(2.99)</td>
</tr>
<tr>
<td>Istanbul</td>
<td>2.30</td>
<td>131</td>
<td>29.7</td>
<td>41.0</td>
<td>1.54</td>
<td>2.69</td>
<td>4.04</td>
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<tr>
<td>London</td>
<td>2.60</td>
<td>107</td>
<td>26.5</td>
<td>320.6</td>
<td>1.87</td>
<td>3.22</td>
<td>4.48</td>
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<tr>
<td>Moscow</td>
<td>3.32</td>
<td>27</td>
<td>7.0</td>
<td>127.4</td>
<td>6.25</td>
<td>7.91</td>
<td>(3.22)</td>
</tr>
<tr>
<td>Paris</td>
<td>3.73</td>
<td>28</td>
<td>6.4</td>
<td>78.5</td>
<td>5.32</td>
<td>6.93</td>
<td>2.62</td>
</tr>
<tr>
<td>Rome</td>
<td>2.95</td>
<td>87</td>
<td>26.4</td>
<td>163.4</td>
<td>2.02</td>
<td>3.67</td>
<td>(3.95)</td>
</tr>
<tr>
<td>Saõ Paolo</td>
<td>3.21</td>
<td>33</td>
<td>10.3</td>
<td>268.0</td>
<td>4.17</td>
<td>5.95</td>
<td>2.72</td>
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<tr>
<td>Sydney</td>
<td>3.33</td>
<td>34</td>
<td>12.3</td>
<td>82.9</td>
<td>2.54</td>
<td>4.37</td>
<td>(4.03)</td>
</tr>
<tr>
<td>Taipei</td>
<td>3.12</td>
<td>74</td>
<td>20.9</td>
<td>186.2</td>
<td>2.42</td>
<td>4.02</td>
<td>(3.74)</td>
</tr>
</tbody>
</table>

$\langle k \rangle$: mean node degree; $\ell_{\text{max}}$, $\langle \ell \rangle$: the maximal and the mean shortest path lengths;

$\langle C \rangle$: normalized clustering coefficient; $\kappa = \langle k^2 \rangle / \langle k \rangle$; $\gamma$: $p(k) \sim 1/k^\gamma$. 

Node-targeted attacks. Choice of an “order parameter”

A node removed: the corresponding PT station ceases to operate, the route splits into two operating pieces.

\[ L\text{-space} \]

\[ S = \frac{N_1}{N} \] and an average inverse mean shortest path length \( \langle \ell^{-1} \rangle = \frac{2}{N(N-1)} \sum_{i > j} \ell^{-1}(i,j) \) as functions of a fraction of removed nodes \( c \).
**Attack strategies**

**Attack strategies: random failures ↔ targeted destructions**

![Graph showing L-space. Largest component size of the PTN of Paris as function of the fraction of removed nodes for different attack scenarios. A superscript i refers to lists prepared for the initial PTN before the attack.](image)

Lists of removed nodes were prepared according to their degree $k$, closeness $C_C$, graph $C_G$, stress $C_S$, and betweenness $C_B$ centralities, clustering coefficient $C$, and next nearest neighbors number $z_2$.

\[
C_C(j) = \frac{1}{\sum_{t \in N} \ell(j, t)}, \quad (1)
\]

\[
C_G(j) = \frac{1}{\max_{t \in N} \ell(j, t)}, \quad (2)
\]

\[
C_S(j) = \sum_{s \neq j \neq t \in N} \sigma_{st}(j), \quad (3)
\]

\[
C_B(j) = \sum_{s \neq j \neq t \in N} \frac{\sigma_{st}(j)}{\sigma_{st}}. \quad (4)
\]
Choice of PTN breakdown indicators: $\ell_{max}$

$L$-space. Recalculated highest degree scenario. Maximal shortest path $\ell_{max}$ and largest connected cluster $S$ for the PTNs of Paris and London. Note the characteristic peaks of $\ell_{max}$ that occur at $c = 0.13$ (Paris) and $c = 0.06$ (London).
A link removed: the route splits into two operating pieces.

\[ BC \text{ link degree: } k_{BC}^{(l)} = k_B + k_C - 2 \]

The size of the largest cluster \( S(c) \) as function of the fraction of removed links. Left: Random link-targeted scenario. Right: Recalculated link-degree attack scenario.

Share of the largest component, mean inverse and maximal shortest path length as function of the removed share \( c \) of nodes for the PTN of London (light green curve) and Paris (dark red curve). Random scenario.

Robustness measure (C.M. Schneider et al., PNAS 108 (2011) 3838):

\[
A = 100 \int_0^1 S(c) \, dc.
\]
### Robustness measures and correlations

<table>
<thead>
<tr>
<th>City</th>
<th>Node-targeted attacks</th>
<th></th>
<th>Link-targeted attacks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RV</td>
<td>$k$</td>
<td>$k^i$</td>
<td>$C_B$</td>
</tr>
<tr>
<td>Berlin</td>
<td>22.7</td>
<td>6.5</td>
<td>7.1</td>
<td>7.3</td>
</tr>
<tr>
<td>Dallas</td>
<td>9.8</td>
<td>3.4</td>
<td>3.6</td>
<td>6.1</td>
</tr>
<tr>
<td>Düsseldorf</td>
<td>25.5</td>
<td>7.5</td>
<td>9.4</td>
<td>8.3</td>
</tr>
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<td>10.0</td>
<td>9.8</td>
<td>6.1</td>
</tr>
<tr>
<td>Istanbul</td>
<td>16.1</td>
<td>4.5</td>
<td>5.0</td>
<td>5.6</td>
</tr>
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<td>29.3</td>
<td>5.5</td>
<td>6.3</td>
<td>8.7</td>
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<td>8.0</td>
<td>8.4</td>
<td>7.8</td>
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<td>13.1</td>
<td>10.7</td>
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<td>22.3</td>
<td>6.6</td>
<td>7.7</td>
<td>7.1</td>
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<tr>
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<td>32.4</td>
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<td>13.6</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Robustness measure $A$ for the PTNs of different cities at different attack scenarios.

**Vulnerability of PT networks**

Percolation cluster: giant connected component (GCC) which for \( N \to \infty \) contains a finite part of a network.

For random network with given \( p(k) \) GCC is present if

\[
\langle k(k-2) \rangle \geq 0
\]

Molloy, Reed’95, Cohen et al.’00, Callaway et al.’00

\[
\kappa(k) = \frac{\langle k^2 \rangle}{\langle k \rangle} = 2 \quad \text{or} \quad \kappa(z) = \frac{\langle z_2 \rangle}{\langle z_1 \rangle} = 1 \quad \text{at} \quad c_{\text{perc}}
\]

\( z_1 \) nearest neighbours

\( z_2 \) next nearest neighbours
Correlation between $A$ and $\kappa$

Attacks on nodes (left) and on links (right). Correlation between $A$ and $\kappa$ for the random scenario. Results for $\kappa^z$ are shown by filled circles, results for $\kappa^k$ are shown by open circles.
Correlation between $A$ and $\langle k \rangle$

Attacks on links. Correlation between $A$ and $\langle k \rangle$ for the random (left) and recalculate degree (right) scenarios.
Correlation between $A$ and $\gamma$

Attacks on nodes. Correlation of $A$ with respect to $\gamma$ for the random (left) and the recalculated node degree (right) scenarios. Filled circles correspond to the PTNs with more pronounced power-law decay of the node-degree distribution.
Cascading effects

A station removed: all routes that service that station cease to operate.

a: A graph with three routes, each shown in a separate color; b: the corresponding bipartite graph - route nodes are depicted as square boxes; c: the weeded graph without dangling station nodes.

The break down of the connected component of the London (yellow) and Paris (blue) PTN under cascading effects. The axis on the left indicates the remaining percentage of the connected part of the network.