Measuring Quantum Tunnelling Dwell Time Using Larmor Precession

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Problem Overview

What is quantum time? Your guess is as good as mine.

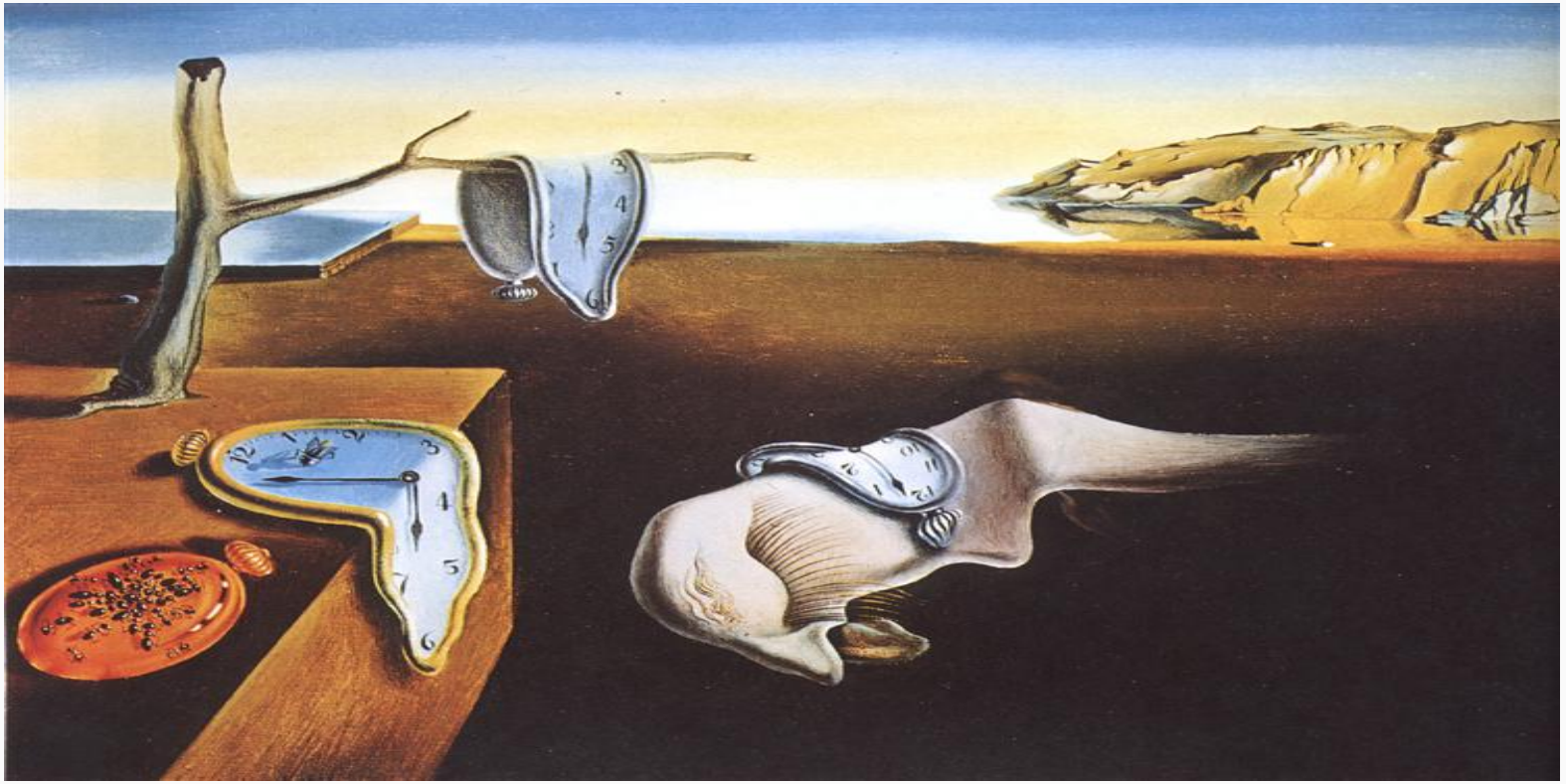
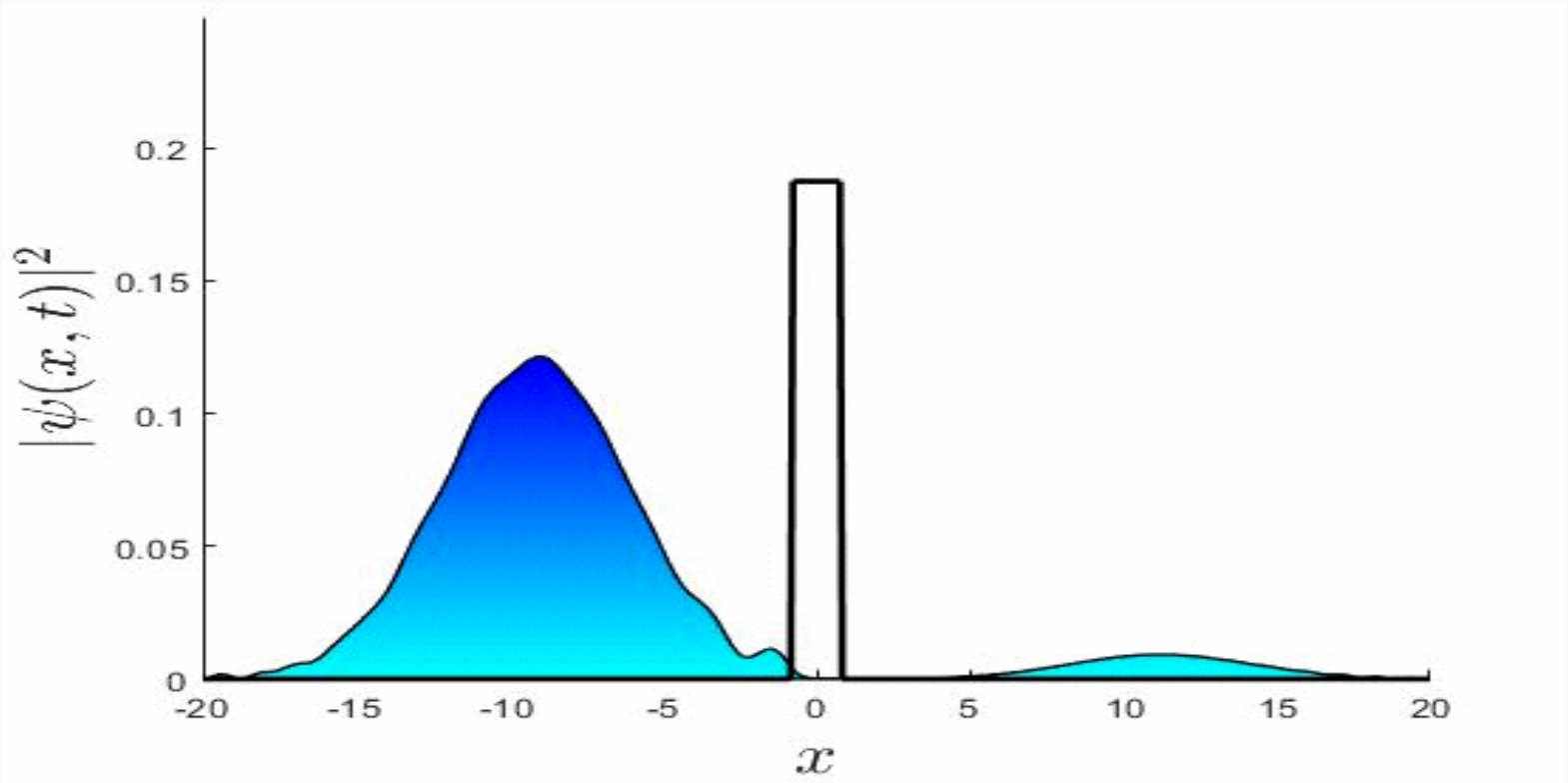


Figure 1: The Persistance of Memory - Dali

Quantum Tunnelling

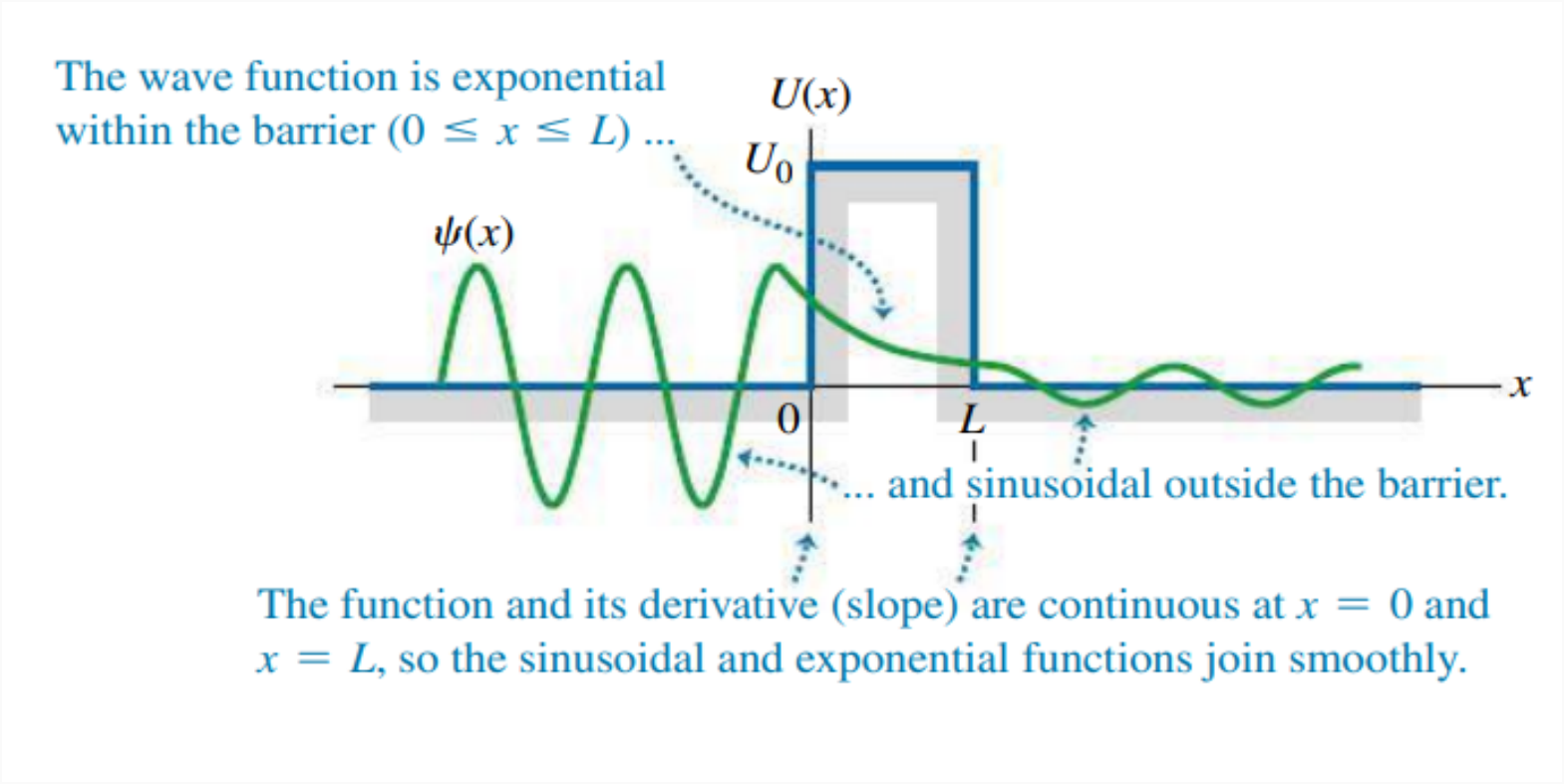
Quantum Tunnelling

Quantum tunnelling is a quantum mechanical phenomenon whereby a wavefunction can propagate through a potential barrier[1].



[1] Alem, *1D FEM analysis of quantum tunneling effect*.

Quantum Tunnelling



Tunneling Diagram[2]

$$\psi_A(x) = Ae^{ik_0x} + Be^{-ik_0x} \quad \psi_B(x) = Ce^{\kappa x} + De^{-\kappa x}$$

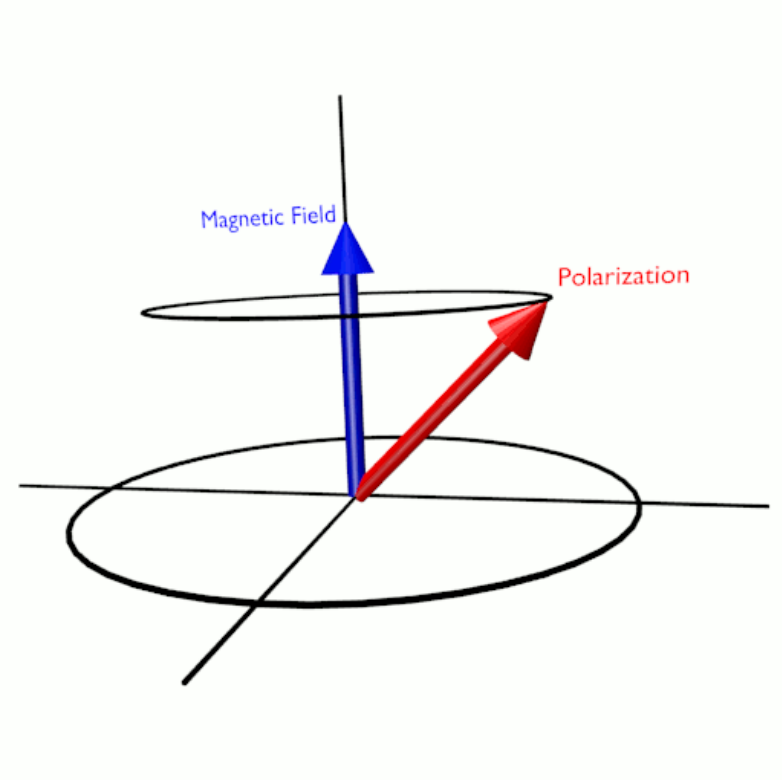
$$\psi_C(x) = Fe^{ik_0x}$$

[2]Young, *University physics : with modern physics*.

Larmor Precession

Classical Larmor Precession

Larmor precession is the precession of the magnetic moment of an object about an external magnetic field.
The frequency of precession is known as the Larmor frequency.[3]



Variation of Angular Momentum **S**:
For a magnetic field $B_0(0, 0, 1)$

$$\frac{d}{dt}S_x = \gamma B_0 S_y$$
$$\frac{d}{dt}S_y = -\gamma B_0 S_x$$

S_x, S_y precess with Larmor Frequency $\omega = \gamma B_0$

[3] Rodriguez, *Polarization Mechanics*.

Quantum Larmor Precession

The quantum mechanical precession is described by a Hamiltonian

$$H == -\gamma S_z B_0$$

$$H = -\gamma B_0 \frac{\hbar}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

There are two eigenvalue energies associated with this Hamiltonian

$$|+\rangle, E_+ = -\gamma B_0 \frac{\hbar}{2}$$

$$|-\rangle, E_- = \gamma B_0 \frac{\hbar}{2}$$

Average Expectation Values

We calculate the x-component expectation value for the particles spin

$$\begin{aligned}\frac{d}{dt}\langle S_x\rangle &= \left\langle \frac{1}{i\hbar}[S_x, H]\right\rangle \\ &= -\gamma B_0 \left\langle \frac{1}{i\hbar}[S_x, S_z]\right\rangle \\ &= \gamma B_0 \langle S_y\rangle.\end{aligned}$$

Repeating for the y-component of the spin recover the classical Larmor formula for the average spin:

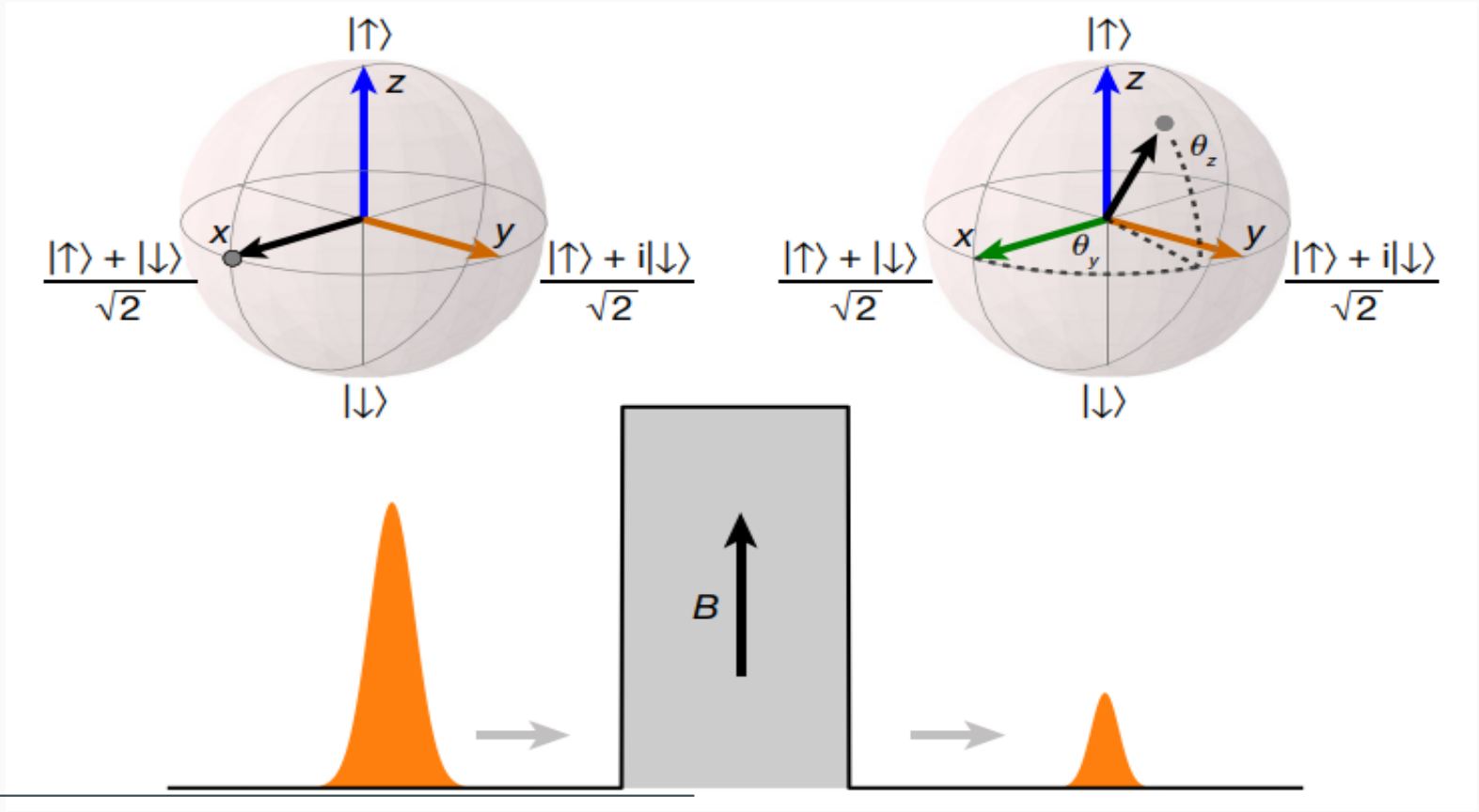
$$\begin{aligned}\frac{d}{dt}\langle S_x\rangle &= \gamma B_0 \langle S_y\rangle \\ \frac{d}{dt}\langle S_y\rangle &= -\gamma B_0 \langle S_x\rangle\end{aligned}$$

Measuring Dwell Time

Measuring Dwell Time

So how can we use this wonderful property?

If a magnetic field is set up in the potential barrier, the particles will undergo larmor precession while quantum tunnelling.[4]



[4] Ramos et al., *Measuring the time a tunnelling atom spends in the barrier.*

Measuring Dwell Time

The stream of particles has a definite spin when entering the potential barrier.
Due to the magnetic field overlayed with the barrier, the particles will undergo precession while tunnelling.
The change in spin is measured on the other side of the barrier after tunnelling.
Therefore, a time value is assigned to the Larmor Frequency in order to quantify this "Dwell Time".

Flaws with Larmor Clock Method

Controversies and Disagreements

The concept of defining a "quantum time" is troublesome.
The Larmor equations determine the average expectation value of the spin over the entire wave function.
However, in measuring only the spin of the particles that have successfully tunnelled through the barrier, the wave function is restricted.
Therefore, the Larmor Equations are no longer being applied over the entire wave function.
It is controversial to assume the Larmor equations exactly describe this situation.

Conclusions

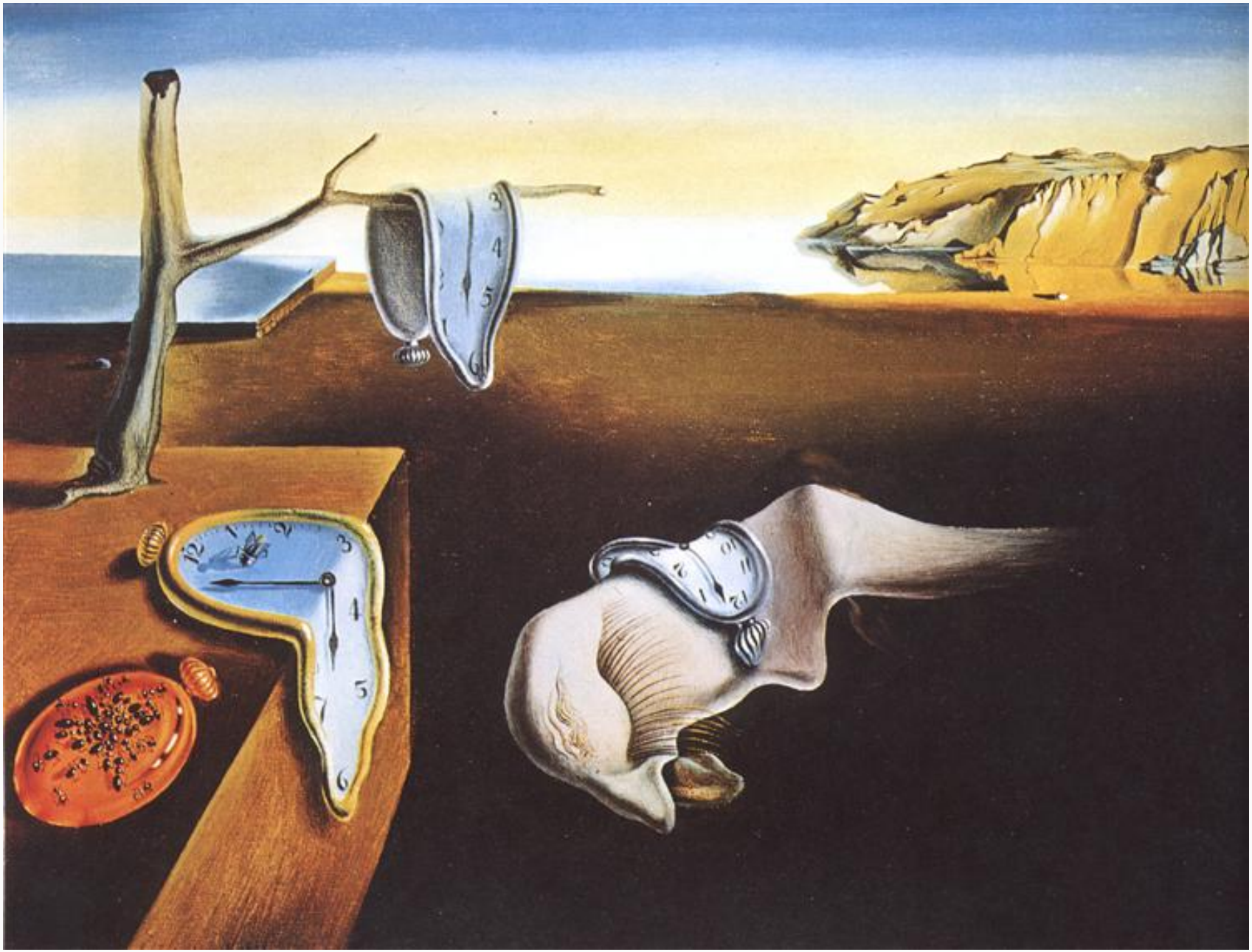
Conclusions

The search to quantify a quantum time is a quest that aims to get to the heart of quantum uncertainty.
It is a current topic which is undergoing a great deal of research, and a great deal of disagreement!
Although time parameters have been assigned in some quantum models, their validity still remains questionable.
More justification is needed to back up claims of defining a concrete time value.

Thanks

Thank you all for listening.

And a special thanks to my supervisor Michael Tuite.



References i

- Alem, Peter van. *1D FEM analysis of quantum tunneling effect*. 2022. URL: https://www.mathworks.com/matlabcentral/fileexchange/82878-1d-fem-analysis-of-quantum-tunneling-effect.
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