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A new methodology to optimise solar energy extraction under cloudy conditions

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ABSTRACT

The orientation and tilt position of the solar panel affect the amount of solar radiation that falls on the panel surface over the course of the day and indeed the year. The choice of tilt angle for a solar panel is fundamental to its efficient operation because incorrectly positioning the solar panel leads to an unnecessary loss in potential power. In the past, much work has been done by authors to determine the optimum tilt angle by applying existing models to their locations. This approach has been successful in climates with the most favourable solar potential, where greater than 90 percent of the solar radiation arrives as direct beam radiation. The accuracy of these models in these locations has been attributed to the low presence of cloud cover and the consequential dominance of the beam radiation portion of the global radiation. Countries located above 45°N however, (Northern Europe), require a different approach to optimising the tilt angle as they receive the least amount of direct radiation with approximately half arriving as diffuse radiation, due to frequent, heavy cloud cover. This paper reviews existing methods and describes a means of predicting the solar radiation in a frequently overcast climate and proposes a method for choosing the optimum tilt angle in such a climate. The effect of different load profiles on the optimum tilt angle is also investigated. The solar radiation model is then used to predict the solar radiation for Cairo, Egypt to show that the model has a global application and is not limited to frequently overcast climates.

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1. Introduction

Two factors responsible for the amount of radiation that is received by a solar panel surface are its orientation and tilt angle. The tilt angle is defined as the angle between the solar panel surface and the horizontal plane. The orientation of the solar panel is also defined with respect to the horizontal plane and it is the angle between the line due south and the projection of the solar panel normal to the surface on the horizontal plane [1]. It has been widely acknowledged that the optimum orientation for a solar panel, in the northern hemisphere, is facing due south. For the choice of the optimum tilt angle, however, there have been several widely diverse proposals. The various schemes used to determine this optimum tilt angle will be described in Section 2. These schemes are separated into two main areas; calculating the tilt angle by latitude angle and maximising the solar radiation falling on the solar panel. The accuracy of these methods has been validated in sunny climates, where the beam portion of the global radiation dominates. In frequently overcast climates, the beam radiation is significantly diminished and the diffuse radiation is prominent. The optimum tilt angle is largely determined by the beam radiation component but in overcast conditions, the diffuse component becomes more significant.

In the absence of measured global radiation data, solar radiation models are used to calculate the radiation. Existing methods such as the Angström Sunshine Hours method, Satellite based methods and Temperature based methods are reviewed in Section 3 with a view to investigating their suitability to frequently overcast climates such as Northern Europe. A new methodology to calculate the solar radiation is proposed which combines hourly observations of cloud conditions with monthly sunshine hours data in order to determine the frequency of clear, partly cloudy and overcast skies. This method provides a more accurate prediction of the diffuse radiation under varying sky conditions. In Section 4.1, the solar radiation values predicted for Cairo, Egypt using this model are compared to the values predicted by the Angström Sunshine Hours method to show that the proposed model has a global application.

The solar radiation model is subsequently used to determine the optimum tilt angle by taking into account the frequency and intensity of the cloud cover. The tilt angle is chosen that optimises the beam radiation on clear days and the diffuse radiation on overcast days. Finally, the effect of different load profiles on the optimum tilt angle will also be discussed.

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2. Review of optimum tilt angle calculations

The tilt angle calculations described in the literature may be broadly classified into two categories; calculation by latitude angle and calculation by maximising the global radiation falling onto the solar panel surface.

2.1. Tilt angle by latitude

The earliest endeavours to calculate the proper choice of tilt angle recommended a yearly optimum tilt angle based primarily on the latitude angle, φ. A brief review of the various schemes is summarised in Table 1. There are clearly wide discrepancies for the proposed optimum tilt angle when the latitude angle is solely used and no definitive value is given. These methods are simple and straightforward but are approximate nevertheless.

2.2. Tilt angle by maximising the solar radiation

A common approach to choosing the tilt angle is to maximise the amount of global radiation falling on the surface of the solar panel. Global radiation has three components; beam, diffuse and reflected. A summary of approaches that maximise the global radiation, or its beam component, falling on the solar panel is given in Table 2. A salient feature of these methods is that they have been adopted in sunny climates where the beam radiation component dominates. A new approach is required in the cloudier climates typical of Northern Europe; this involves taking the frequency and intensity of cloud cover into account and this is described in the next section.

2.3. Importance of the tilt angle under cloudy conditions

The previously described methods for tilt angle calculations have focused on areas located between the latitudes of 5°N and 40°N, placing them in the most favourable belt for receiving solar radiation. The radiation received across the globe varies extensively and is unevenly dispersed due to variables such as the time of day and the season, and the prevailing atmospheric conditions, which are determined by cloud coverage and level of pollution. In the northern hemisphere, the greatest amount of radiation is received by locations situated between latitudes 15°N and 35°N, for example Egypt, with greater than 90 percent arriving as direct beam radiation. The next favourable belt lies between 15°N and the equator which includes Central America. Countries located between the latitudes 35°N and 45°N, such as Spain or Turkey, experience significant seasonal variations resulting in less radiation received. The least favourable locations are situated beyond 45°N, for example Northern Europe, as they receive the least amount of direct radiation. Many countries in Northern Europe are located in this belt such as Ireland, England, Norway and Sweden. Approximately half of the radiation arrives at the surface as diffuse radiation due to frequent heavy cloud cover. For this reason, methods that choose the tilt angle by maximising the beam or extraterrestrial radiation are unsuitable, as they do not take atmospheric conditions into account. For a climate susceptible to overcast skies, where the beam radiation is eliminated, a new approach is necessary that takes into account the frequency of clouds. The proposed methodology combines hourly observations of cloud conditions with monthly sunshine hours data in order to determine the frequency of clear, partly cloudy and overcast skies in order to calculate the solar radiation. Using these solar radiation values and knowledge of cloud conditions, the tilt angle can be chosen that optimises the available solar radiation between the beam radiation on sunny days and the diffuse radiation on overcast days. The starting point for these tilt angle methods is the calculation of solar radiation and its constituent components. The various approaches for estimating solar radiation will be described in the next section.

Table 1
Tilt angle by latitude angle.

Author	Recommended tilt angle	Application
Duffie & Beckmann [2]	$(\varphi+15^\circ)\pm15^\circ$	These methods
Heywood [3]	$\varphi-10^\circ$	were designed for space
Lunde [4]	$\varphi\pm15^\circ$	and solar water
Chinnery [5]	$\varphi+10^\circ$	heating. The minus
Löf and Tybout [6]	$\varphi+(10^\circ\rightarrow30^\circ)$	sign refers to the summer
Garg [7]	$\varphi+15^\circ$	season and the positive
	$\varphi-15^\circ$	sign is for the winter.
	0.9φ	

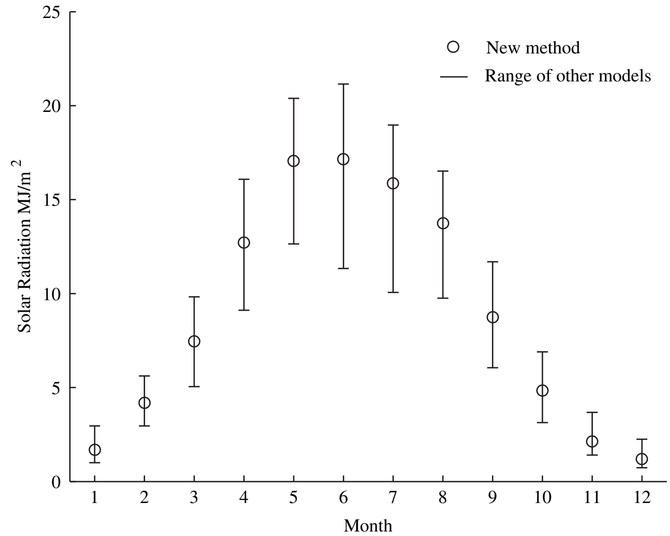

Fig. 1. Comparison of proposed method to commonly used methods for estimating solar radiation in Ireland.

3. Review of current methods to estimate solar radiation and its components

The design of solar energy systems depends on the availability of accurate localised solar radiation in order to correctly predict the system performance. Much work has been done to estimate the global solar radiation such as the Angström Sunshine Hours method, Satellite based methods and Temperature based methods. There are several approaches to find the Angström empirical constants a and b and these will be described in Section 4.1. The three approaches under review are summarised in Table 3. The new methodology for the tilt angle optimisation is based on the sun hours to total day length (n/N) ratio used in the Angström Sunshine Hours method and therefore, it will be described in more detail.

3.1. Sunshine hours to day length ratio (n/N)

When global radiation data is not available to derive the empirical constants for the Angström Sunshine Hours method, the sunshine hours data may be used in an alternative way to calculate the solar radiation. The ratio of n/N, first introduced by [16] and [17] (Table 3), is used as the basis for the new solar radiation model and therefore warrants further explanation. The ratio of n/N may be used to predict the percentage of sunny days while $(N-n)/N$ corresponds to the percentage of cloudy days [24]. The airmass value, A, determines the path length travelled by the direct beam

Table 2
Tilt angle by maximising solar radiation.

Author	Methodology	Comment
Gunerhan [8] (Turkey) Kern [9] (South Africa)	• Maximises extraterrestrial radiation falling on the solar panel. • Maximises beam radiation falling on the solar panel.	• The method maximising the extraterrestrial radiation does not take into account the attenuation of the solar radiation as it passes through the earth's atmosphere. • These methods may be effective in sunny climates with little cloud cover. • Unsuitable in climates with cloudy conditions because they do not include the effects of varying diffuse or reflected radiation. •The latitude angle is recommended as the tilt angle.
Hartley [10] (Spain) Yakup [11] (South East Asia) Elminir [12] (Egypt)	• Maximises global radiation falling on the solar panel. • These methods use measured values of horizontal global and diffuse radiation in a model to predict the solar radiation on a tilted plane. • The models used for predicting the solar radiation on a tilted plane include work by Liu and Jordan, [13] and Temps-Coulson, [14]	• The accuracy of these models has been accredited to the low presence of cloud cover in these areas and the dominance of the beam radiation portion of the global radiation. • The accuracy of the models was assessed using the mean bias error (MBE), root mean square error (RMSE) and the t-statistic. The equations for these are given in the Appendix.
Elsayed [15]	• Investigates the optimum tilt angle for a solar collector as a function of the number of glass covers, latitude angle, ϕ, monthly average clearness index, K_T, (defined as the ratio of the global radiation to the extraterrestrial radiation), month, and ground reflectivity. • The correlation to predict the optimum value of tilt angle as a function of clearness index is given as: $\beta_{opt} = \left(6 - 4.8\overline{K}_T + 0.86\overline{K}_T^{0.27}\varphi + 0.0021\varphi^2\right) + \left(31\overline{K}_T^{0.37} + 0.094\overline{K}_T^{0.46}\varphi + 0.00063\overline{K}_T^{-0.17}\varphi^2\right)\cdot\cos\left[\frac{360}{365}(D+11.5)\right]$ (1) D Day number	• Elsayed observed that as the clearness index decreased and all other parameters were kept constant, the optimum ilt angle decreased.

radiation through the atmosphere for every hour and is a function of the sun's altitude angle, α. The value for the airmass is one when the sun is directly overhead and the solar radiation has the shortest path to travel through the atmosphere. Airmass values determined throughout the day are used to calculate the hourly beam component of the solar radiation [25], and are calculated as follows:

$$A = \frac{1}{\cos(90-\alpha) + 0.50572(96.07995-(90-\alpha))^{-1.6364}} \quad (3)$$

This value increases as the sun approaches the horizon. The direct beam component, incident on a plane perpendicular to the sun's rays, $I_{B,N}$, may be determined from the hourly airmass values [26].

$$I_{B,N} = 1.353 \times 0.7^{A^{0.678}} \quad (4)$$

The value 1.353 corresponds to the value of the solar constant and the 0.7 is the amount of solar radiation that is transmitted through the atmosphere (70%). The direct beam component, $I_{B,N}$,

Table 3
Existing methods of estimating the solar radiation.

Approach	Methodology	Drawback
Angström Sunshine Hours Method Angström [16] Prescott [17]	• The measured number of sunshine hours is used to estimate the monthly average daily global solar radiation on a horizontal surface. $\frac{H}{H_o} = a + b\frac{n}{N}$ (2) H: Monthly global radiation H_o: Monthly average daily extraterrestrial radiation n: Monthly average sunshine hours N: Maximum possible day length measured in hours a, b: Empirical constants	• The sunshine duration may lead to an overestimate of cloudy conditions [18]. • a and b are dependent on local conditions gathered from global radiation data. They are derived from long term data fitting, thus limiting the area of application to a specific location. • a and b are commonly given constant values despite the fact that they vary appreciably throughout the year due to seasonal atmospheric changes.
Satellite Based Methods Hammer [19]	• Global horizontal radiation is estimated from the pixel value of the satellite image. • Clear sky radiation is calculated for a given location and then a cloud index is derived from the pixel value of the satellite images. • Reducing the clear sky radiation by the cloud cover derives the ground global radiation.	• Discrepancies occur on clear winter days because the satellite imagery is unable to distinguish between clouds and fog, frost or snow [20] • The diffuse radiation is underestimated under thin and scattered clouds because the satellite imagery neglects the solar radiation that is reflected back off the ground or clouds. • Satellite derived data is not accurate for site specific applications due to the interpolation techniques used to derive the solar radiation data over vast regions. • Not indicative of the actual climate as the local conditions differ from place to place.
Temperature Based Methods Bristow [21] Hargreaves [22] Supit [23]	• Global radiation is estimated using a model relating the atmospheric transmittance and the difference between the maximum and minimum daily temperatures. • Model extended to include cloud cover.	• Unsuitable for wet environments in Northern Europe • Unsuitable in Ireland because temperature variations are not the result of cloud effects. The Gulf Stream is a more dominant temperature moderator and masks the effects of cloud cover.

determines the values perpendicular to the sun's rays and must be adjusted for the cosine effect to determine the beam radiation on a horizontal surface. This effect changes the intensity of the solar radiation falling on the solar panel and is determined by the angle of incidence, which is the angle between the sun's rays falling on the solar panel surface and the solar panel normal.

The beam component is then increased by 10% to account for the diffuse component on a clear day [26] to give the total global radiation on a plane perpendicular to the sun's rays, I_{GN}.

$$I_{GN} = 1.1 I_{B,N} \tag{5}$$

On a cloudy day, all the incoming radiation is assumed to be diffuse with an intensity approximately equal to 20% of the direct beam component given by equation (4) [24]. In this way, the average number of clear days and cloudy days is determined from the sun hours data ratio.

This method described the number of hours that do not contribute to the sun total and is not indicative of the level of cloudiness. The fact that there is no sun may be due to a bright overcast or a dark overcast. This range corresponds to sky cloud coverage of approximately 60% to 90%. This is a wide range when it is considered that this is the difference between a solar panel operating at 10% or 40% full intensity.

The methods described in this Section and Section 3.1 lack accuracy in determining the diffuse component of the global radiation and lack localised application. The next section describes a methodology to overcome these limitations.

4. Proposed methodology for estimating the solar radiation

Monthly hours of sunshine duration with hourly observations of cloud conditions are readily available in Europe and the US and may be combined to give an improved estimate of the solar radiation because localised data is used. Since the methodology includes localised conditions, accuracy is improved for site specific applications without the need for expensive solar radiation measuring equipment.

In the Sunshine Hours Ratio (n/N) method previously described in Section 3.1, inaccuracies occurred as the level of cloudiness was not taken into account. With the proposed methodology a distinction can be made between clear, bright and dark overcast days. Clouds are one of the main factors that affect the amount of solar radiation hitting the earth's surface and are usually the largest source of error in theoretical models. General descriptions are used by meteorologists to describe sky conditions: clear, intermediate, partly cloudy and overcast. It is accepted that on a clear day there are few clouds and none of the sun is blocked. On a totally overcast day, typically only ten percent of the sun's intensity reaches the surface of the earth. In intermediate conditions, clouds cover 15–60% of the sky while cloudy skies are typically 50–90% covered. The meteorological station, Metcheck, provides hourly cloud conditions for every day of every month, for the years 2003–2006 for Ireland and the United Kingdom. Similar stations exist that provide details for the rest of the world. Therefore, for every month of the year the number of hours with reported clear, intermediate, partly cloudy and overcast skies can be determined. Ireland has a particularly changeable climate and is susceptible to partly cloudy and overcast skies more than half the time. An issue arises for the days with an observed amount of cloud, as the relative position of the sun and the clouds is not noted, therefore it is not known whether the sun is blocked in these conditions. The monthly sunshine hour's data is also available for all months in the same period. The frequency of occurrence of clear, bright and dark overcast skies can be determined with the following steps.

a. The number of observed sunny hours is subtracted from the monthly sun hours data, n. This gives the number of intermediate and partly cloudy sky conditions that do not block the sun and therefore contribute to the monthly sun hours data.
b. The number of intermediate and partly cloudy sky conditions that contribute to the sun hours data is subtracted from the total number of observed intermediate and partly cloudy hours. This gives the number of hours of bright overcast skies.
c. Finally, the number of hours of observed dark overcast skies is determined. These sky conditions do not contribute to the number of sun hours.
d. The number of hours for each type of sky condition is expressed as a ratio of the maximum number of day length hours per month to determine the frequency of clear, bright and dark skies for that month.
e. This procedure is repeated for a number of years and an average of the frequency of clear, bright and dark skies is determined to gain typical values.

The hourly airmass values, direct beam component and global radiation on a clear day are then calculated per equations (3)–(5). For a bright overcast day, manufacturers typically suggest that 60% of the global radiation reaches the surface and on dark overcast days as little as 10% of the radiation gets through. These values are obtained by measuring the performance of the solar panel under bright and overcast conditions and comparing to the output expected under full sun intensity [27]. Subsequently, the global radiation on bright and dark overcast days is split into its direct beam and diffuse components using a diffuse fraction correlation developed by [28]. This expression estimates the diffuse fraction as a function of the clearness index, K_T, and the altitude angle, α, and is described by the following equations:

$$K_d = 1.020 - 0.254K_T + 0.0123 \sin\alpha \quad (K_T \le 0.30) \tag{6}$$

$$K_d = 1.4 - 1.749K_T + 0.0177 \sin\alpha \quad (0.30 < K_T < 0.78) \tag{7}$$

$$K_d = 0.486\, K_T - 0.182 \sin\alpha \quad (K_T \ge 0.78) \tag{8}$$

The global radiation on a clear, bright and overcast day is then combined with the cloud conditions frequency of occurrence in order to find the total global radiation. The monthly average daily global radiation falling on a horizontal plane, I_{GTot}, predicted by the proposed method for Ireland is compared with the values obtained by other commonly used methods, as described in Section 3, (Table 3). These results are summarised in Fig. 1. The bars show the range of solar radiation predictions. The models compared are the Angström Sunshine Hours method, (the a and b constants for Ireland were provided by [29]), the Satellite based method, the Temperature based model and the method using the Sunshine Hours Ratio (n/N). Evidently, during the winter months, there is less available solar radiation and so the span of values obtained is small while in the summer months, the range is wider.

The monthly average hourly radiation values on a horizontal surface, I_G, are determined from the monthly average daily radiation values I_{GTot}, as shown in Fig. 1, using a correlation proposed by [30].

$$I_G = r I_{GTot} \tag{9}$$

The ratio r is defined as:

$$r = \frac{\pi}{24}(x + y\cos\omega)\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \frac{\pi}{180}\omega_s\cos\omega_s} \tag{10}$$

$$x = 0.409 - 0.5016\sin(\omega_s - 60) \quad (11)$$

$$y = 0.6609 - 0.4767\sin(\omega_s - 60) \quad (12)$$

ω: hour angle in degrees for the midpoint of the hour under interest; ω_s: Sunset hour angle.

The values of the monthly average hourly global radiation are shown in Fig. 2. The month of June is shown because it has the longest day length and displays the most variation in the values obtained from the different solar radiation models.

4.1. Validation of solar radiation model

The proposed solar radiation methodology was applied to two locations; Galway, Ireland and Cairo, Egypt. In this section, it is shown that the method may be adapted and used successfully in sunnier locations. Cairo, Egypt is located at latitude 30°N, placing it in the most favourable belt for receiving solar radiation. The radiation values predicted by the proposed solar radiation model are compared with those of other methods to determine its accuracy. The solar radiation measurements for Cairo were taken from the World Radiation Data Centre (WRDC) from 1972 to 1988. Five approaches based on the Angström Sunshine Hours method were specifically developed for Cairo and are used for comparison. These are methods A–E in Table 4. Methods A, B and C were developed by [31]. Method D was developed by [32] and Method E was developed by [33]. These five methods determined the Angström Sunshine Hours constants, a and b, using regression analysis between the ratio of the global radiation, H, to extraterrestrial radiation, H_o, and the ratio of the sun hours, n, to day length, N, (Equation (2)) using measured global radiation values from meteorological stations in Cairo. The differences between the methods may be summarised as follows. In methods A, B and C, the constants were derived under time periods of varying lengths. In Method A, the constants were calculated from regression analysis between H/H_o and n/N for long periods of each month. In Method B, the constants were calculated from the monthly values of H/H_o for each year and the average of these values was used. Methods D and E also followed this method, varying only in the duration of periods of data studied, five and three years respectively. Method C takes the same approach as in Method B, but uses a nonlinear equation previously validated in [34] to determine the monthly mean daily global radiation. To compare the accuracy of the calculated values of the various models, the absolute

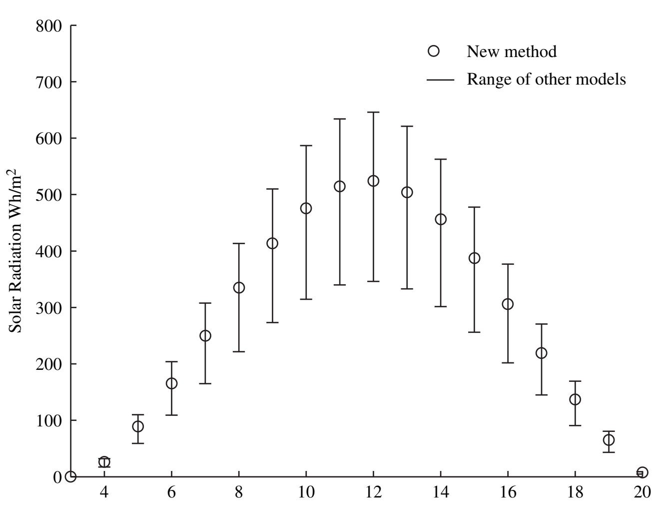

Fig. 2. Monthly average hourly radiation of the solar radiation models for June.

Table 4
Comparing the accuracy of the proposed solar radiation method with Angström Sunshine Hours method for Cairo.

Solar radiation method	Annual % error	Error range	MBE	RMSE	t-Statistic
A [31]	4.56	2.08–8.46	0.163	0.9	0.612
B [31]	4.46	0.73–9.8	0.149	0.824	0.611
C [31]	3.95	0.81–8.18	−0.09	0.808	0.372
D [32]	4.34	0.6–10.73	0.305	0.892	1.206
E [33]	3.9	0.4–7.29	−0.32	0.806	1.434
New proposed method	3.34	0.37–6.6	−0.28	1.01	1.14

percentage of the deviation of the predicted values of global radiation from the measured data was calculated. Table 4 shows the range of the errors values for each method. The average annual percentage error indicates that the new proposed method gives the best estimation for the global solar radiation averaged over the specified time period. Stone [35] recommends that the t-statistic be used in conjunction with the mean bias error (MBE) and root mean square error (RMSE) to more reliably assess a model's performance. The smaller the value of the t-statistic, the better the performance of the model. The monthly mean predicted solar radiation was compared with the monthly mean measured radiation. The statistical errors were calculated for every month and an average was obtained for each solar radiation model. These values are also given in Table 4. The MBE, which provides information on the long-term performance of the model, identifies how closely the predictions agree with measured data. A negative MBE shows that the proposed model underestimates the correct value, while a positive MBE overestimates the value. A small MBE is desirable. RMSE provides information on the short-term performance of a model by allowing a term by term comparison of the actual deviation between the estimated and the measured values.

Method C has the lowest value of MBE, meaning that this model has less variation between the measured and predicted values. The negative value shows that this model tends to underestimate the solar radiation. The RMSE value of Method C is also at the lower range of the methods reviewed. The low values of MBE and RMSE combine to give the lowest value of t-statistic, making it the most accurate solar radiation model for Cairo over the studied time period. Method E has the lowest RMSE value, but combined with the largest MBE value gives the biggest t-statistic overall. It can be seen that Methods D and E have the largest values of MBE and t-statistic, despite the fact that similar methodologies are used in Methods A, B, and C. The difference in accuracy may be explained by the varying time periods used in the derivation of the Angström Sunshine Hours constants, a and b. Methods A, B, and C used 16 years of global radiation data, while Methods D and E used five and three years respectively. From the statistical parameters, it may be reasoned that using longer time periods to derive the Angström constants is necessary to establish the climatic patterns of a region, using averaging to eliminate possible changes in atmospheric conditions due to effects such as pollution.

The proposed method is the next best model, performing better than methods D and E, thus proving the new proposed method may be used to estimate the solar radiation with easily obtained parameters and without the need for expensive measuring equipment. The proposed model has a negative MBE value shows that the model underestimates the solar radiation available. As previously stated in the proposed methodology, it is accepted that for a bright overcast day, 60% of the global radiation reaches the surface and on dark overcast days as little as 10% of the radiation gets through. Given that Cairo has a very different climate to Ireland, with high levels of pollution, the accuracy of the proposed model may be improved by including the average transmittance characteristics of localised cloud cover. This could be achieved by observing the available global radiation on bright and dark days and comparing the global radiation to values typical of clear skies.

5. New methodology for the choice of optimum tilt angle

In Section 4, a new method for determining the solar radiation was presented. In this section, a method for choosing the optimum tilt angle is described. Under cloudy skies, some or all of the direct beam radiation, which largely determines the optimal angle for solar panels, is obscured. Therefore it is important to distinguish between direct and diffuse radiation for a particular site.

A high frequency of diffuse radiation may result in a lower optimum tilt angle than expected. On an overcast day, where the direct beam is eliminated and the global radiation is predominately diffuse, maximum radiation would be received by a horizontal solar panel. This concept is echoed in Elsayed [15], (Table 2). It was observed that the optimum tilt angle decreased with decreasing clearness index. The new methodology incorporates the effect of frequently overcast skies on the choice of optimum tilt angle. The diffuse radiation falling on the solar panel is predicted using the Perez Anisotropic model described in [36]. This model splits the diffuse radiation falling on a tilted solar panel into three different components; the circumsolar radiation surrounding the sun's disk,

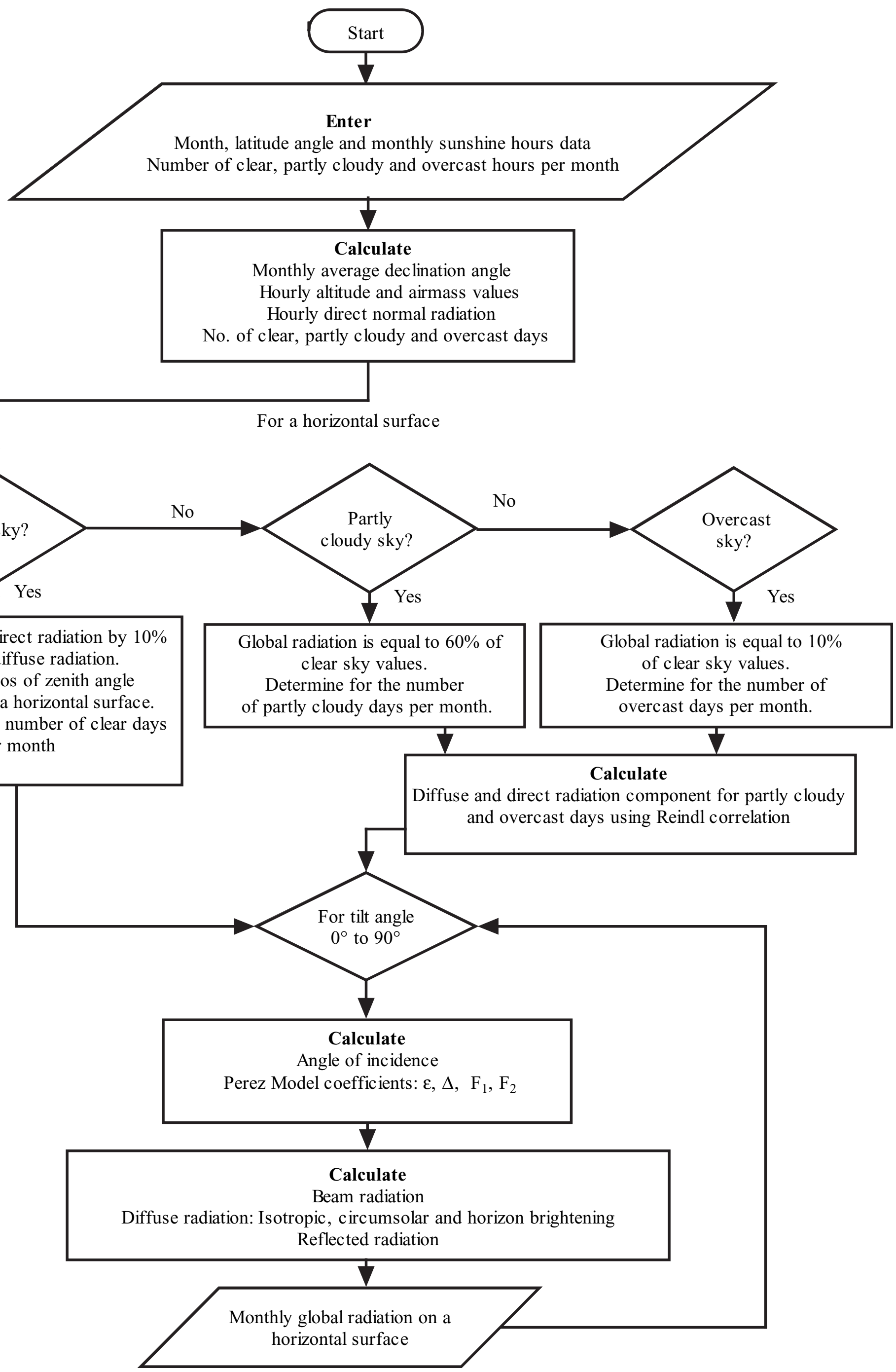

Fig. 3. Flowchart of the methodology for solar radiation and optimum tilt.

the horizon brightening radiation and the isotropically distributed diffuse radiation from the rest of the sky. The diffuse radiation, I_{DT} is given as:

$$I_{DT} = I_{DH}\left[\left((1-F_1)\frac{(1+\cos\beta)}{2}\right) + F_1\frac{a_p}{b_p} + F_2\sin\beta\right] \quad (13)$$

I_{DH}: diffuse radiation on a horizontal plane; F_1: circumsolar brightness coefficient; F_2: horizon brightening coefficient; β: Solar panel tilt angle; $a_p = \max(0°, \cos\theta)$ where θ is the angle of incidence; $b_p = \max(\cos 85°, \cos\theta_z)$ where θ_z is the zenith angle.

The coefficients F_1 and F_2 are dependent on the sky conditions as determined by the sky clearness, ε, and the sky brightness, Δ; these equations are given in [36]. The sky clearness, ε, expresses the change in sky conditions from totally overcast to a clear sky; and the sky brightness value, Δ, represents the thickness of the cloud cover.

The flowchart for the methodology of determining the solar radiation and the tilt angle is shown in Fig. 3. The solar radiation received by a solar panel is investigated under varying tilt angles, from 0° to 90° in steps of 1°, and varying cloud conditions. The tilt angle is chosen that maximises the solar radiation falling on the solar panel. Taking into account the frequency of cloudy skies, the tilt angle is chosen that optimises the beam radiation on clear days and the diffuse radiation on overcast days. For Ireland at a latitude angle of 53°, this angle is 33°. The results for the optimum tilt angle for Ireland are compared with the tilt angles obtained by maximising the solar radiation assuming clear skies using the geometric factor [12] and Elsayed's method. Elsayed's method is used for comparison as it takes into consideration the characteristics of the atmosphere by incorporating the clearness index. This comparison is shown in Fig. 4. It can be seen that the choice of tilt angle is lower than would be chosen under clear skies. This enables better interception of diffuse radiation on overcast days. Tang and Wu [37] concluded that the Elsayed correlation was inappropriate for locations with a low clearness index as the correlation used to estimate the diffuse component was unsuitable in such climates. This is evident for Ireland in the months of November and December when the highest deviations from the tilt angle suggested by Elsayed occur.

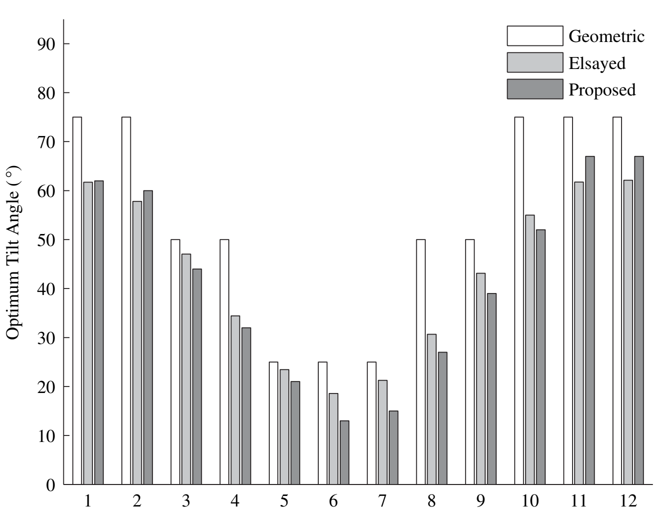

Fig. 4. Comparison of the proposed tilt angles with the geometric factor and Elsayed's method.

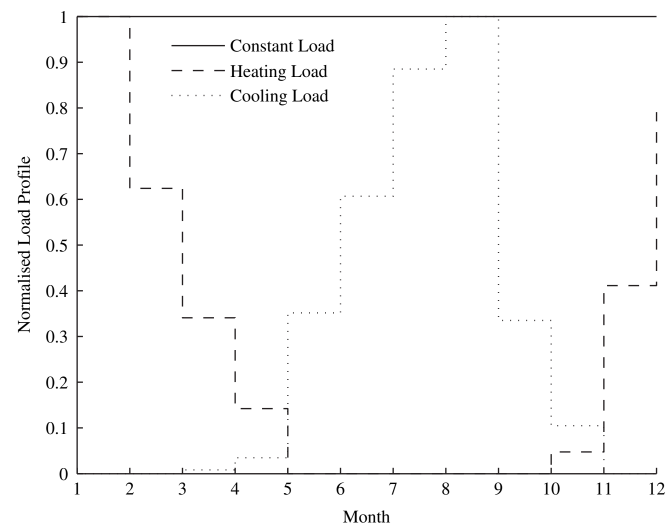

Fig. 5. Normalised yearly loads.

6. Effect of load on the optimum tilt angle

In Section 5, the methodology for choosing the optimum tilt angle by maximising the solar radiation received by a solar panel was presented. However, in some cases, this may not be the objective of choice. For example, for grid-connected systems in locations that do not use net metering, the power generated that exceeds the load requirements is wasted because the excess energy is not credited by the utility and therefore has no benefit to the consumer. Also, in the case of off-grid systems, the tilt angle that maximises the solar radiation received over the year may not be able to supply the actual load requirements of the system. Therefore, a much more worthwhile objective is to match the availability of the solar radiation to the load requirements. In this section, the effect of the load profile on the choice of optimum tilt angle is investigated. The optimum tilt angle is chosen as the angle that matches the solar radiation availability with the load demand. The choice of optimum is based on maximising the optimisation factor, F_{opt}. This is given as [38]:

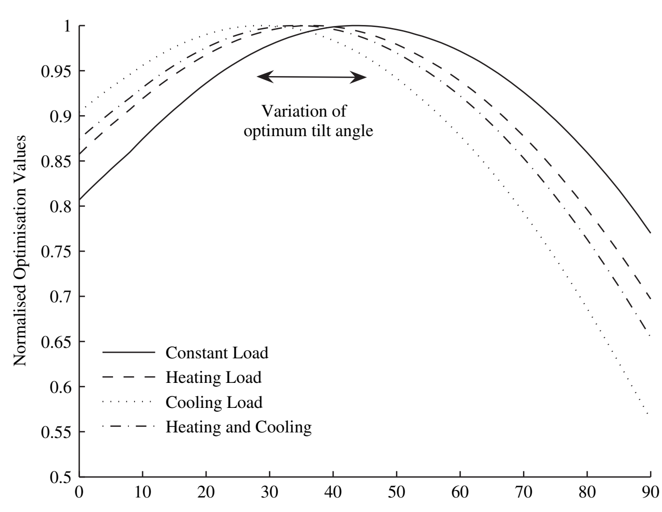

Fig. 6. Variation of the normalised optimisation factor as a function of the tilt angle.

$$F_{\mathrm{opt}} = \frac{G_{\mathrm{T}}}{\mathrm{Stdev}(\beta)} \tag{14}$$

G_{T}: monthly average solar radiation falling on a solar panel tilted at an angle, β; Stdev(β) Standard deviation of the difference between the normalised load curve and the normalised solar radiation at angle, β.

Four load profiles were considered; a constant load, heating load, cooling load and a combined heating and cooling load. The normalised values of these yearly loads are shown in Fig. 5. The optimum tilt angle occurs when the optimisation factor is at a maximum. The variation of the normalised optimisation factor for each as a function of the tilt angle is shown in Fig. 6. The optimum tilt angles for Galway, Ireland for a constant load, heating load, cooling load and a combined heating and cooling load are 44°, 37°, 30° and 35° respectively.

7. Conclusions

A new method for choosing the optimum tilt angle for a solar panel has been presented which is particularly suitable, but not limited to, climates susceptible to frequently overcast skies. This method can be applied to any location with knowledge of the monthly sunshine duration and hourly cloud observations as shown in the prediction of solar radiation values for Cairo, Egypt and Galway, Ireland. The optimum tilt angle for maximising the solar radiation over a year in Ireland was shown to be lower than that obtained assuming clear skies, as this enables better utilisation of diffuse radiation. The effect of the load on the choice of the optimum tilt angle was also investigated which determined the tilt angle by finding the best match between the solar radiation availability and the yearly load requirements. Possible future work was also suggested to address the shortcomings identified in the paper, such as including the average transmittance characteristics of localised cloud cover in order to increase the accuracy of the solar radiation model.

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Appendix

Statistical indicators are used to determine the accuracy of the performance of the solar radiation models to the measured values of global radiation. The predicted values are assessed by the mean bias error (MBE) and the root mean square error (RMSE). The mean bias error is given by

$$\mathrm{MBE} = \frac{1}{n}\sum_{i}^{n} d_i \tag{A1}$$

The root mean square error is given by

$$\mathrm{RMSE} = \sqrt{\left(\frac{1}{n}\sum_{i}^{n} d_i^2\right)} \tag{A2}$$

n: number of data pairs; d_i: Difference between the ith predicted and the ith measured data.

In conjunction with the MBE and RMSE another statistical indicator, the t-statistic, is proposed in [35] as a means of evaluating and comparing solar radiation models. The t-statistic is given as:

$$t = \sqrt{\left(\frac{(n-1)\mathrm{MBE}^2}{\mathrm{RMSE}^2 - \mathrm{MBE}^2}\right)} \tag{A3}$$

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