Light transmission from a randomly rough dielectric diffuser: theoretical and experimental results

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We report numerical and experimental observations of a new transmission effect in rough dielectric interfaces.

Recent experiments with light scattered from highly sloped random rough surfaces\textsuperscript{1,2} and the development of new numerical methods of study\textsuperscript{3,4} have produced a renewed interest in scattering effects and in the mechanisms that produce them, in both metallic and dielectric surfaces.\textsuperscript{5-10}

We report the numerical prediction and experimental demonstration of a new transmission effect in a semi-infinite dielectric material when light is incident from a semi-infinite vacuum upon the rough surface that separates both media.

Because of the limited power of computer calculations in terms of both speed and memory, the surface profiles under study are one dimensional: the random height depends on one transverse coordinate only, being constant along the other coordinate. The experiments have also been performed on specially fabricated one-dimensional surfaces in order to accommodate the observations to the available theory.

Numerical Monte Carlo calculations for the scattered intensities have been carried out from samples with rms deviation \( \sigma \), normal statistics, and a Gaussian correlation function, with profile correlation length \( T \). The surfaces are generated by means of a procedure identical to that described in Refs. 3 and 7, namely, each sample consists of a portion of a sequence of \( \sim 10^5 \) random numbers with the desired normal probability density and statistical parameters \( \sigma \) and \( T \). The length \( L \) of the samples is typically \( L = 40 \lambda \) (\( \lambda \) being the wavelength); 220 sampling points are taken for each sample. Both the field and its normal derivative are calculated on the surface by using the extinction theorem.\textsuperscript{11} The corresponding equations are similar to those already used by Maradudin \textit{et al.} in Refs. 4 and 10, with the sole exception that each sample is assumed to be illuminated by a plane wave instead of a beam (i.e., no tapering is made). Then those boundary values of the fields and their derivatives on the surface are introduced into the far-zone expression of the field so the scattered intensity can be found. Finally these scattered intensities are averaged over 240 samples. At a given angle of incidence and a certain surface record of length \( L \), two sets of scattered intensities are obtained by considering the results for \( \theta_0 \) and those for \( -\theta_0 \). This doubles the effective number of samples. Calculations were done on a CDC Cyber 180/855 computer. Unitarity of the normalized reflected and transmitted intensities at the dielectric interface, as well as the convergence of the results when the number of samples is increased, is taken as a criterion of numerical consistency.

The angular distribution of mean intensity in the far zone of light transmitted into the dielectric, after being scattered at the rough surface separating it from vacuum, is plotted in Fig. 1. The dielectric constant is \( \Re(\epsilon) = 1.991 \) [\( \Im(\epsilon) \approx 0 \)], where \( \Re \) and \( \Im \) denote real and imaginary parts, respectively, and the surface parameters are \( \sigma = 1.86\lambda \) and \( T = 4.69\lambda \). The sum of the reflectance \( R \) and the transmittance \( T \) for each angle of incidence (\( 0^\circ, 20^\circ, 40^\circ, \) and \( 60^\circ \), respectively) is shown in Fig. 1 (solid curves, \( p \) polarization; dashed curves, \( s \) polarization). We have marked the direction of refraction that would correspond to a plane interface (dotted vertical lines) and the straight-through direction (small marks at the left of the vertical lines at the tops of the figures). The remarkable effect observed is the deviation from the refraction direction of the mean scattered intensity of light transmitted into the dielectric. A narrow distribution peaked closer to the straight-through direction appears. The roughness makes the angular distribution of light act as though there were no different refractive index on the other side. Moreover, this peak tends to grow and to become narrower as the angle of incidence increases.

We believe that the mechanism that produces this effect can be understood within the diagrammatic approach used by the authors of Refs. 1, 2, 4, 9, and 10. Because of the slope of the surface, the local angle of incidence decreases as the overall angle of incidence increases. On the other hand, since the material is highly transparent, little light is thrown back into vacuum each scattering event, so double-scattering contributions are negligible in comparison with those of single scattering. These single-scattering contributions tend to broaden the distribution of transmitted light, but the dominant angle of light transmission is observed to be greater than it would be if there were no roughness (Snell's law), and it is close to the straight-
through direction: this is due to the aforementioned lower local angle of incidence. In the case of nonnegligible multiple-scattering contributions (greater refractive index), since the second hit on the surface would contain almost all local angles of incidence the resulting angular distribution should be expected to spread over a wide solid angle failing to peak clearly in one direction. Calculations made by increasing $\epsilon$ have confirmed this point.

The effect reported in this Letter is observed neither on surfaces whose radius of curvature is much larger than $\lambda$ (this is the general criterion of validity of the Kirchhoff approximation for perfect conductors) nor with very low $\sigma$ and $T$ (say, one tenth of $\lambda$). However, as this effect is essentially due to single scattering, we believe that a calculation based on the Kirchhoff approximation should give a nearly correct solution; recent preliminary computations confirm this belief (this suggests that the range of validity of the Kirchhoff approximation for dielectric surfaces is much broader than accepted for perfect conductors).

In order to verify the findings of the numerical transmission calculations, we fabricated a dielectric diffuser in silicone rubber to produce a slab without a tilt angle. The diffuser considered here is a dielectric replica of the diffuser investigated in reflection by Sant et al.\textsuperscript{6}

Using an experimental rig essentially identical to that described by O'Donnell and Méndez,\textsuperscript{2} with the exception of implementing a transmission geometry, we took transmission data for $0^\circ$, $20^\circ$, $40^\circ$, and $60^\circ$ incidence for both $s$ and $p$ polarizations; the results are shown in Fig. 2. The data were converted by Snell's law of refraction (for the plane back face of the sample) to represent light transmission within the medium. The data have not been normalized in an absolute manner, but they are relatively comparable.

For $0^\circ$ incidence the transmission peak does not appear to have been detected; the precise reason for this is not known but is thought to be misalignment. The refractive index of the silicone rubber at $\lambda = 633$ nm has been more accurately determined to be $n = 1.411$ since it was reported in Ref. 6. One would therefore expect total internal reflection to occur at a plane dielectric interface for angles of incidence beyond $\arcsin(1/1.411) \approx 45.1^\circ$; this is, in fact, observed for $40^\circ$ and $60^\circ$ incidence, where the detected intensity abruptly falls to zero at $\approx -45^\circ$. The dashed curve on each graph of Fig. 2 corresponds to the refraction angle at which the transmitted light would propagate if there were no front-surface roughness.

The agreement between the experimental and numerically calculated results is excellent; particular attention is drawn to the angular position of peak trans-

![Fig. 1](image-url)
Fig. 2. Measurements of transmission envelopes scattered from a dielectric surface illuminated at a wavelength of 0.633 μm: 1/e correlation length 2.97 ± 0.05 μm, rms height 1.18 ± 0.13 μm, and refractive index 1.411.

mission and angles at which the scattered intensity falls to zero. Although for 60° incidence only a small portion of the transmitted light is detected, agreement is still good.

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