Multiple scattering from rough dielectric and metal surfaces using the Kirchhoff approximation

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Abstract. The Kirchhoff double-scatter method for calculating the intensity distribution scattered from a rough surface is extended to dielectric and metal surface materials. The material properties are contained in the Fresnel reflection coefficients only. It is shown that the results agree well with calculations using the exact method for a surface of Gaussian statistics with standard deviation of height \( \sigma = 1.93 \lambda \) and 1/e correlation length \( \tau = 5.02 \lambda \).

1. Introduction

In a previous paper [1] it was shown that using the Kirchhoff approximation (KA) and including shadowing, it was possible to derive equations for the single- and double-scatter contributions to the intensity distribution scattered from a randomly rough surface. In [1], the simplest situation was considered—that of a perfectly conducting surface. As expected, the double-scatter term showed the enhanced backscatter effect [2-4], with a peak approximately equal to twice the background in the backscatter direction. This agrees with the simple ray picture of the scattering process for which rays and their time-reversed partners add coherently in the backscatter direction and incoherently in other directions.

A comparison of the perfect conductor calculations with experimental distributions of the scattered light from a gold-coated surface with Gaussian statistics showed good agreement at low angles of incidence (up to about 20° from normal) but poor agreement at higher angles [1]. The same trend is seen in comparisons of exact calculations using the extinction theorem with experimental values [4], although for perfectly conducting surfaces the calculated intensities for the extinction and Kirchhoff double-scatter methods agree very well [5]. This suggests that the Kirchhoff method has included the main physical processes that produce enhanced backscatter.

In this paper the results of the method for the more general cases of scattering from a dielectric and a general conductor (metal) are reported. In section 2 the theory is briefly outlined and in section 3 the results are shown and compared with exact calculations and experimental results.

2. Theory

The starting point is the two-dimensional Helmholtz integral equation for a scattered field (the surface profile is constant along the y-direction):

\[
E_s(x, z) = \frac{1}{4i} \int \left( E_s(x', z') \frac{\partial H_0^{(1)}(kr)}{\partial n} - H_0^{(1)}(kr) \frac{\partial E_s(x', z')}{\partial n} \right) \, dx',
\]

(1)
where \( E(x', z') \) is the total field at the point \((x', z')\), \( r = [(x-x')^2 + (z-z')^2]^{1/2} \), \( H_0^{(1)}(kr) \) is the zeroth order Hankel function of the first kind and \( ds' \) is an element of the surface.

The KA approximates the total field on the surface as the sum of the incident field plus the reflected field at that point:

\[
E(x, z) = (1 + R)E_i(x, z),
\]

\[
\frac{\partial E(x, z)}{\partial n} = i(1 - R)k \cdot n E_i(x, z),
\]

where \( R \) is the planar reflection coefficient at that point, depending on the local incidence angle; \( E_i(x, z) \) is the field incident at that point; \( \mathbf{k} \) is the incident wavevector; and \( n \) is the outward normal to the surface at the point \((x, z)\). Since \( R \) is the planar reflection coefficient, we immediately have a condition for the validity of the approximation—the surface must be locally flat or, as is usually written, the radius of curvature of the surface must be large compared with the wavelength.

Substituting (2) and (3) into (1) and performing a small amount of mathematics, following Beckmann and Spizzichino [6], the standard single-scatter solution is obtained:

\[
E_s(\theta) = -\left( \frac{2}{\pi kr} \right)^{1/2} \frac{\exp i \varphi}{2} \frac{1 + \cos(\theta + \theta_i)}{\cos \theta + \cos \theta_i} \int \! R(x', z') \exp i(\mathbf{k}_1 - \mathbf{k}) \cdot \mathbf{R} \; dx',
\]

where the reflectivity is a function of \( x \), since the local angle of incidence varies along the surface. In equation (4), \( \theta_i \) is the incidence angle, \( \theta \) is the angle of scatter (measured positive in the opposite direction to the incident angle) and \( \varphi \) is a phase factor depending only on \((x, z)\) and not on \((x', z')\). To obtain this expression we have assumed an incident plane wave of unit amplitude. Shadowing effects must somehow be included in this method to have a physically realistic result. This is done by multiplying the integrand in (4) by incidence and scatter shadow functions

\[
S(x', z') = \begin{cases} 
1 & \text{if } (x', z') \text{ is illuminated}, \\
0 & \text{if } (x', z') \text{ is not illuminated}, 
\end{cases}
\]

\[
S'(x', z') = \begin{cases} 
1 & \text{if } (x', z') \text{ is visible}, \\
0 & \text{if } (x', z') \text{ is not visible}.
\end{cases}
\]

These represent geometrical or straight line shadow functions and so are an approximation to the true effect of shadowing.

The derivation of the double-scatter contribution involves two terms. The first is the field scattered from one point on the surface to another point, also on the surface,

\[
E_s(x_2, z_2) = -\frac{1}{4i} \left( (1 + R_1) \left( m_1 \frac{k(x_2 - x_1)}{r_{12}} - \frac{k(x_2 - x_1)}{r_{12}} \right) H_1^{(1)}(kr_{12}) \right.
\]

\[
- (1 - R_1) i k (m_1 \sin \theta_i + \cos \theta_i) H_0^{(1)}(kr_{12}) \bigg) E_i(x_1, z_1),
\]

where subscript 1 represents the first point and subscript 2 the second. The normal derivative of the Hankel function is given by

\[
\frac{\partial H_0^{(1)}(kr_{12})}{\partial n} = \left[ n_x \frac{k(x_2 - x_1)}{r_{12}} + n_z \frac{k(x_2 - x_1)}{r_{12}} \right] H_1^{(1)}(kr_{12}).
\]
The second term is the field scattered into the space above the surface from every combination of first and second points:

\[
E^s_2(\theta) = -\left(\frac{2}{\pi kr}\right)^{1/2} \frac{\exp i\phi}{4} \int_{r_1} \int_{r_2} S^{s}_{12} \left( (1 + R_2) k(m_2 \sin \theta - \cos \theta) E_0(x_2, z_2) \right.
\]

\[
+ i (1 - R_2) \frac{\partial E_0(x_2, z_2)}{\partial n_2} \right) \exp ik \cdot \mathbf{R}_2 \, dx_1 \, dx_2,
\]

(7)

where we have included another shadowing function to describe shadowing between points on the surface:

\[
S_{12} = \begin{cases} 
1 & \text{if } (x_2, z_2) \text{ is visible from } (x_1, z_1), \\
0 & \text{if } (x_2, z_2) \text{ is not visible from } (x_1, z_1),
\end{cases}
\]

and substituted

\[
S'_{12} = S(x_1, z_1) S_{12} S'(x_2, z_2).
\]

In equation (7) we have used equation (2) a second time to write that the total field at the second point is the field from the first point plus its reflection \((R_2)\). This can be seen to be reasonable if it is noted that the condition on the radius of curvature depends only on the surface profile and so will be true for all subsequent interactions if it is true for the first. The validity of the parameters used here was discussed in the previous paper. This method is the same as the iterative Kirchhoff solution [7, 8] but with the inclusion of the shadow functions giving the effect of shadowing explicitly rather than implicitly as in the iterative method [8].

Equations (5) and (7) together form the double-scatter contribution. The simplest case to consider is when the material is perfectly conducting. Then the reflection coefficients \(R\) are either 1 or \(-1\) for \(p\) (TM) and \(s\) (TE) polarizations, respectively, and we are left with only one term as already discussed. For the more general case we end up with the sum of four terms. In all the cases considered, the normal derivative \(\partial E(x_2, z_2) / \partial n_2\) is approximated in the following way:

\[
\frac{\partial E_0(x_2, z_2)}{\partial n_2} = \frac{1}{4i} \left( (1 + R_1) \left( m_1 \frac{k(x_2 - x_1)}{r_{12}} - \frac{k(x_2 - z_1)}{r_{12}} \right) \frac{\partial H_0^{(1)}(kr_{12})}{\partial n_2} \right.
\]

\[
\left. - (1 - R_1) i k(m_1 \sin \theta_1 + \cos \theta_1) \frac{\partial H_0^{(1)}(kr_{12})}{\partial n_2} \right) E_i(x_1, z_1).
\]

(8)

In (8) we need to use

\[
\frac{\partial H_0^{(1)}(kr_{12})}{\partial n_2} = \left[ \frac{1}{2} \left( H_0^{(1)}(kr_{12}) - H_2^{(1)}(kr_{12}) \right) \right].
\]

The material dependence of these equations is contained solely in the Fresnel reflection coefficients \(R\). For scattering from a dielectric surface of refractive index \(n\) in air \((n = 1)\), the reflection coefficients for \(p\) and \(s\) polarizations are [9]

\[
R_p = \frac{n \cos \theta_1 - \sqrt{\left(1 - \frac{\sin^2 \theta_1}{n^2}\right)}}{n \cos \theta_1 + \sqrt{\left(1 - \frac{\sin^2 \theta_1}{n^2}\right)}},
\]

(9)
To find the scattered field we substitute either (9) or (10), depending on the polarization, into (5) and (7). This case of the perfect dielectric is reasonably straightforward. A more complicated situation occurs when the case of light scattered from an interface between a dielectric and air is considered when the light is incident from the dielectric side of the boundary. In this case the Fresnel coefficients are [9]

\[ R_p = \frac{\cos \theta_1 - n\sqrt{1 - n^2 \sin^2 \theta_i}}{\cos \theta_1 + n\sqrt{1 - n^2 \sin^2 \theta_i}}, \]
\[ R_s = \frac{n \cos \theta_1 - \sqrt{1 - n^2 \sin^2 \theta_i}}{n \cos \theta_1 + \sqrt{1 - n^2 \sin^2 \theta_i}}. \]

Then, when

\[ n^2 \sin^2 \theta_i > 1, \]

the reflectivities are complex. This occurs when the critical incidence angle \( \theta_c \) is reached and we have total internal reflection

\[ \theta_c = \sin^{-1}\left(\frac{1}{n}\right). \]

The final case considered is that of the general conductor or metal which involves a complex refractive index \( n \rightarrow n + i\kappa \). In this case the reflectivities are [9]

\[ R_p = \frac{(n^2 \cos^2 \theta_1 - q^2 \cos^2 \gamma + (\kappa^2 \cos^2 \theta_1 - q^2 \sin^2 \gamma)\cos \theta_1 + q \cos \gamma)^2 + (\kappa \cos \theta_1 + q \sin \gamma)^2}{(n \cos \theta_1 + q \cos \gamma)^2 + (\kappa \cos \theta_1 + q \sin \gamma)^2}, \]
\[ R_s = \frac{\cos^2 \theta_1 - (nq \cos \gamma - \kappa q \sin \gamma)^2 - (\kappa q \cos \gamma + nq \sin \gamma)^2}{(n \cos \theta_1 + (nq \cos \gamma - \kappa q \sin \gamma))^2 + (\kappa q \cos \gamma + nq \sin \gamma)^2}, \]

where

\[ q^2 = \left(1 - \frac{(n^2 - \kappa^2)}{(n^2 + \kappa^2)^2} \sin^2 \theta_i\right)^2 + \left(\frac{2n\kappa}{(n^2 + \kappa^2)^2} \sin^2 \theta_i\right)^2, \]
\[ \tan 2\gamma = \frac{\left(1 - \frac{(n^2 - \kappa^2)}{(n^2 + \kappa^2)^2} \sin^2 \theta_i\right)^2}{2n\kappa}. \]

It should be noted that the contribution from light paths that traverse inside parts of the surface are not taken into account. It is believed that such terms are small due to the requirement for large angles of deviation of refraction to direct light back into the space above the surface.
3. Results

The Kirchhoff method equations have been discretised in the usual way (assuming that values of parameters remain constant over a suitably small range of variables) and programmed into a Sun 4 computer. A surface of length $30\lambda$ was discretized into 200 segments giving a segment length of $0.151\lambda$. Computation times were 7 minutes per frame for the dielectric cases and 10 minutes per frame for the metal with approximately 600 frames averaged for each graph. Computational results are shown in figure 1 with experimental results [10] shown in figure 2 for comparison. The surface parameters are: standard deviation of height $\sigma = 1.18\ \mu m$, correlation length $\tau = 2.97\ \mu m$, with a wavelength of $0.633\ \mu m$ (He-Ne red) and a refractive index of $n = 1.41$. The scattered energy as a function of incidence angle for both methods is given in table 1.

The Kirchhoff and exact methods agree quite well both for scattered energy and the intensity distribution up to $-30^\circ$ incidence. In both the s and p cases the Kirchhoff scattered intensity distribution is rising at scatter angles near $+90^\circ$ and for p-polarization the energy scattered is twice its value for the extinction case. The cause of this may be that the effect of light paths that pass through the material are becoming important, or the approximation used for the shadow functions, i.e. straight line or geometric shadow functions, is less valid when incidence shadowing starts to have an effect. The second reason is supported by the fact that for the perfectly conducting case the unitarity was from 6 to 9% greater than unity. There is reasonable agreement with the shape of the experimental curves, although note that the experimental values have not been normalized. However, from the Kirchhoff results it can be seen that the double-scattered energy is an order of magnitude down on the single scatter for s-polarization (TE) and several orders of magnitude down for p-polarization (TM). This means that scattering from this dielectric surface is mainly a single-scatter effect, so that even though enhanced backscatter can be seen in the double-scatter curves, the effect on the total scatter curve is very small. This is probably because only a small part of the energy is reflected at each interaction point on the surface.

Assuming that in this single-scatter regime the dominant term from each part of the surface is the specular term it can be shown that the minimum in the p-polarization case is directly related to the Brewster angle. Figure 3 shows the situation where one of the discretized surface facets is illuminated at the Brewster angle and so will not scatter p-polarized light. Then $\theta_s$, the scatter angle, is given by

$$-\theta_s = \theta_i + 2\theta_B,$$

since $\theta_i$ is negative. For $n = 1.41$, the Brewster angle is

$$\theta_B = \tan^{-1}(1.41) \approx 55^\circ.$$

Thus at $\theta_i = -30^\circ$, $\theta_s = -80^\circ$ and at $\theta_i = -60^\circ$, $\theta_s = -50^\circ$, which agrees reasonably well with the observed angle at which the minimum occurs. Another feature of the curves which can be explained using the single-scatter model is that the s and p curves have the same value at backscatter for all incidence angles. If the specular term is dominant then the backscatter results from parts of the surface which are normal to the incident direction. The Fresnel coefficients for normal reflections have the same modulus but different sign for s and p, so the scattered intensities are the same in the backscatter direction.
Figure 1. Scattering cross-section as a function of angle for p (right) and s polarization, 0° (top), -30° (middle) and -60° incidence from a dielectric surface $n=1.41$ with $\sigma=1.18 \mu m$ and $\tau=2.97 \mu m$. Kirchhoff total (solid line), single (oooo), double (+ + + +) and extinction calculation (dotted line). Backscatter is marked by the vertical dashed line.
Figure 2. Experimental curves for scattering from a dielectric as in figure 1: s (upper curve) and p (lower curve) polarizations.

Table 1. Scattered energy as a function of incidence angle (dielectric).

<table>
<thead>
<tr>
<th>Incidence angle</th>
<th>Kirchhoff single</th>
<th>Kirchhoff double</th>
<th>Kirchhoff total</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s-polarization (TE)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.031</td>
<td>0.0012</td>
<td>0.031</td>
<td>0.035</td>
</tr>
<tr>
<td>−30°</td>
<td>0.037</td>
<td>0.0028</td>
<td>0.038</td>
<td>0.043</td>
</tr>
<tr>
<td>−60°</td>
<td>0.081</td>
<td>0.0022</td>
<td>0.079</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>p-polarization (TM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.018</td>
<td>0.0001</td>
<td>0.0180</td>
<td>0.020</td>
</tr>
<tr>
<td>−30°</td>
<td>0.018</td>
<td>0.0001</td>
<td>0.0181</td>
<td>0.018</td>
</tr>
<tr>
<td>−60°</td>
<td>0.043</td>
<td>0.0002</td>
<td>0.043</td>
<td>0.018</td>
</tr>
</tbody>
</table>
The results for the inverse dielectric case for the Kirchhoff method with s-polarization and the same material as above \((n=1.41)\) are shown in figure 4. The scattered energy is shown in table 2.

In this case, the double scatter forms a much larger part of the total scattered energy so that a strong enhanced backscatter peak is expected. Indeed, this is seen at low angles of incidence but the peak dies away quickly so that by \(-20^\circ\) it has almost disappeared completely. The steep slope of the double-scatter curve between \(0^\circ\) and \(20^\circ\) gives this effect.

The peak in the single-scatter curves at large negative angles can be explained in the same way as the Brewster angle effect above. The scatter angle corresponding to a local angle of incidence equal to the critical angle is given by

\[
-\theta_s = \theta_i + 2\theta_c, 
\]

where the critical angle \(\theta_c\) is given by

\[
\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \approx 45^\circ, 
\]

giving \(\theta_s = -70^\circ\) for \(\theta_i = -20^\circ\). This angle should give the start of the region of total internal reflection. Shadowing counteracts this effect by blocking light scattered at high angles, causing the reflectivities to come down again toward \(-90^\circ\).

The final case considered is that of scattering from a metal, in particular gold, at \(\lambda=0.633\ \mu m\), with the refractive index \(n=0.167+i3.149\) [11]. Previous comparisons of experiment and theory have used calculations from perfect conductors, so it would be useful to find the effect of finite conductivity on the scattered intensity. The scattered energy for two angles of incidence is given in table 3.
Multiple scattering and the Kirchhoff approximation

Figure 4. As figure 1, for scattering from a dielectric-air interface from the dielectric side: 0° (top), -10° (middle) and -20° incidence angles.

Table 2. Scattered energy (inverse dielectric).

<table>
<thead>
<tr>
<th>Incidence angle</th>
<th>Kirchhoff single</th>
<th>Kirchhoff double</th>
<th>Kirchhoff total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.058</td>
<td>0.037</td>
<td>0.095</td>
</tr>
<tr>
<td>-10°</td>
<td>0.070</td>
<td>0.043</td>
<td>0.114</td>
</tr>
<tr>
<td>-20°</td>
<td>0.097</td>
<td>0.050</td>
<td>0.148</td>
</tr>
</tbody>
</table>
Table 3. Scattered energy for two different angles of incidence (gold).

<table>
<thead>
<tr>
<th>Incidence angle</th>
<th>Kirchhoff single</th>
<th>Kirchhoff double</th>
<th>Kirchhoff total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>s-polarization (TE)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.811</td>
<td>0.105</td>
<td>0.902</td>
</tr>
<tr>
<td>-40°</td>
<td>0.847</td>
<td>0.075</td>
<td>0.872</td>
</tr>
<tr>
<td><strong>p-polarization (TM)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.783</td>
<td>0.112</td>
<td>0.882</td>
</tr>
<tr>
<td>-40°</td>
<td>0.829</td>
<td>0.069</td>
<td>0.849</td>
</tr>
</tbody>
</table>

Figure 5. As figure 1, for a metal surface at 0° (top) and -40° incidence. Kirchhoff total (solid line), single (oooo) and double (+ + + +) renormalized to have unit area under the total curve. The dotted line is the Kirchhoff total for the same surface as a perfect conductor.
Multiple scattering and the Kirchhoff approximation

These values of scattered energy could be used as a check on the parameters of the surface material, allowing a check on whether there is an unexpected effect present in the scattering process, for example, if the metal coating is too thin or if there is some granulation of the surface material. Here, to compare the perfect conductor and metal results, the two cases are normalized to the same area under the graph (as has been done on previous experimental results) and compared directly. Figure 5 shows the results. There is very little difference in the metal and perfectly conducting curves, showing that finite conductivity has very little effect on the scattered intensity distribution.

4. Conclusions

Using the Kirchhoff double-scatter contribution it has been shown that scattering from a particular dielectric surface is mainly a single-scatter effect when the light is incident from the air side, but when the incidence is from the dielectric side much more light is double scattered. We believe that the reflectivities at each interaction are very low in the first case and total internal reflection gives relatively more double scattering in the second. The results agree well with exact calculations using the extinction theorem, except at higher angles of incidence where it is thought that the approximation used for shadowing becomes less valid. Scattering from a metal surface was compared with scattering from a perfect conductor and found to agree well. This shows that the effect of finite conductivity, at least for very good conductors, is purely to reduce the scattered energy and not to alter the scattered intensity distribution.

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References