Effects of retinal scattering in the ocular double-pass process

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The validity of double-pass wave-front measurements in the eye is reviewed analytically and computationally. A mathematical description of the scalar optical field in the exit pupil plane after an ocular double-pass process is presented. With the help of this description, the relationship between the phase information losses and the statistical properties of retinal scattering is demonstrated. © 2001 Optical Society of America

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1. INTRODUCTION

Phase information loss that is due to the double-pass process in optics is a well-known problem. It is present when trying to estimate the optical quality of the eye and is described by Artal et al. for the double-pass point-spread function (PSF). The problem is revisited in this paper, the interest being the wave front in the pupil plane.

In this case, the scattering nature of the surface that marks the end of the first pass and the beginning of the second pass will determine if the propagating wave front emerging from the complete process retains the phase delay of the first and second passes or only that of the second one. This becomes crucial when trying to measure the single-pass optical quality in a double-pass configuration. If the information of both passes is kept, then an incorrect estimate of the single-pass wave front is obtained; but if the phase of the first pass is lost, the estimated wave front is only that of the second pass.

A mathematical formalism to describe the double-pass process is introduced. The information losses in a double-pass configuration are described. Emphasis is placed on the statistical properties of the scattering surface, and the particular case of the eye is considered.

2. DOUBLE-PASS PROCESS

A double-pass process can be described as follows: A certain wave of known characteristics travels through a perturbative medium. At the end of this medium, a surface scatters the propagating wave and causes it to travel back through the same medium. At the end of this process, the wave has been affected in the same way twice while traveling back and forth, and, in addition, it has been scattered. In a double-pass configuration, it is usually possible to measure only the features of the wave emerging from this back-and-forth process. In the particular case of the eye, one is interested in how the wave, or in particular its wave front, is affected after having traveled only once through the perturbative medium. This information has to be extracted from the measured features of the double-pass wave.

To analyze the double-pass process, we use the unfolded geometry shown in Fig. 1. The system forms a real and inverted image of the object, as the eye does. For the first pass of the light through the optical system, the object plane is denoted \((x_o, y_o)\), the pupil plane is \((\xi_1, \eta_1)\), and the image plane is \((x_1, y_1)\); for the second pass, the object plane is \((x_1, y_1)\), the pupil plane is \((\xi_2, \eta_2)\), and the final image plane is \((x_2, y_2) = (x_o, y_o)\). Planes \((x_o, y_o)\), \((x_1, y_1)\), and \((x_2, y_2)\) are conjugates. All the phase delays are assumed to be introduced in planes \((\xi_1, \eta_1)\) and \((\xi_2, \eta_2)\). This assumption implies that the pupil planes are being illuminated by aberration-free wave fronts and that all the phase delays are introduced by a thin phase plate existing within the pupil plane. As long as isoplanicity is not an issue, this assumption is valid.

Let \(\mathcal{L}(\xi_j, \eta_j) = |\mathcal{L}(\xi_j, \eta_j)| \exp[-i(2\pi\lambda) W(x_j, y_j)]\) be the generalized pupil function of the \(j\)th pass in a double-pass process in the pupil plane \((\xi_j, \eta_j)\). The amplitude impulse response of the \(j\)th pass at the \(j\)th image plane \((x_j, y_j), j = 1, 2\), is given by

\[
A_j(x_j, y_j) = \frac{\exp \left[ \frac{ik}{2s_j} (x_j^2 + y_j^2) \right]}{i\lambda s_j} \int \exp \left[ -\frac{2\pi}{\lambda s_j} (x_j\xi_j + y_j\eta_j) \right] \mathcal{L}_j(\xi_j, \eta_j). \tag{1}
\]

Let the object in plane \((x_o, y_o)\) be a point source. The image in plane \((x_1, y_1)\) is the amplitude impulse response \(A_1(x_1, y_1)\) given by Eq. (1). This complex object interacts with the scattering surface in this plane; in the case of the eye, this scatterer is the retina. This interaction can be described by a phase screen \(R(x_1, y_1, t)\), which includes all the phase delays and the changes in amplitude introduced by the scatterer as a function of position and time. It is a function of time so as to later account for the movements of the eye, and it is assumed to change on a time scale of \(T\). For simplicity, the changes in \(A_j(x_j, y_j)\) that are due to the changes in \(\mathcal{L}_j(\xi_j, \eta_j)\) (for instance, movements of the eye) are supposed to be negligible.
The instantaneous wave front of the optical field is described by

$$A'_1(x_1, y_1; t) = A_1(x_1, y_1)R(x_1, y_1; t).$$  \hspace{1cm} (2)

After propagation, the optical field at the exit pupil plane is proportional to the Fourier transform of $A'_1(x_1, y_1; t)$. In this plane, it interacts with the generalized exit pupil $L_2(\xi_2, \eta_2)$. This process is described by the following relation:

$$L_{dp}(\xi_2, \eta_2; t) = C \text{FT}[R(x_1, y_1; t)A_1(x_1, y_1)]L_2(\xi_2, \eta_2)$$

$$= C[\text{FT}[R(x_1, y_1; t)] \otimes \text{FT}[A_1(x_1, y_1)]]$$

$$\times L_2(\xi_2, \eta_2)$$

$$= C[R(\xi_2, \eta_2; t) \otimes L_1(-\xi_2, -\eta_2)]$$

$$\times L_2(\xi_2, \eta_2),$$  \hspace{1cm} (3)

where $C$ is a constant of proportionality, $\text{FT}[\cdot]$ denotes the Fourier transform of $[\cdot]$, and $R$ is the Fourier transform of $R$. The negative sign of the coordinates in $L_1(-\xi_2, -\eta_2)$ is explained as follows: The Fourier transform applied twice to a function is equal to the original function inverted in the new coordinate space:

$$\text{FT}[\text{FT}[f(\xi_1, \eta_1)]] = f(-\xi_2, -\eta_2).$$  \hspace{1cm} (4)

Relation (3) is valid for a static scatterer or for a moving scatterer observed during a period of time $t_o \ll T$. In general, $L_{dp}(\xi_2, \eta_2; t)$ can be described by the complex function

$$L_{dp}(\xi_2, \eta_2; t) = |L_{dp}(\xi_2, \eta_2; t)|$$

$$\times \exp \left[-i \frac{2\pi}{\lambda} W_{dp}(\xi_2, \eta_2; t)\right],$$  \hspace{1cm} (5)

where $W_{dp}$ is the instantaneous wave front of the optical field. This function $W_{dp}$ is the quantity estimated from data obtained with a wave-front sensor such as a Shack–Hartmann sensor. Note, from Eq. (3), that $L_{dp}$ is a function of $R$. In the case of a nondeterministic scattering surface (such as the retina), $R$ is a random variable. Its mean value and standard deviation are determined by the statistical properties of the scattering surface. As an example consider Fig. 2. In it, the dashed curve represents the wave front $W_1$ of the first-pass optical field, and the solid curve is the wave front $W_{dp}$ of the instantaneous optical field after a symmetric double-pass process. The final double-pass wave front follows roughly the shape of the single-pass wave front, but it has a random modulation. Consequently, the instantaneous wave-front measurement is expected to be subject to a relatively large random error that is due to scattering. In Sections 3 and 4, we will come back to this figure and how it was generated. For now we will consider the general case of a nonstatic scatterer.

If the scatterer is moving, a long time observation of the optical field can be used to obtain the average value of different quantities; for instance, the average intensity of the scattered field at the pupil plane or the average of the reconstructed wave fronts. However, a measurement of the average optical field $\langle L_{dp} \rangle$ cannot be directly obtained.

The principle of operation of a Shack–Hartmann sensor has been described elsewhere; see, for example, Ref. 5. In this type of sensor, a lenslet array produces an array of spots over an intensity detector, such as an array of quaddells or a CCD array. When an intensity detector is used, the image obtained from a long time exposure is equivalent to the image generated from the sum of many short time exposures. In the context of a Shack–Hartmann sensor, it means that the average position of each spot produced by the lenslet array onto the detector must be the same, independent of the averaging method utilized, i.e., a single long exposure or the average of many short exposures.

The wave front is reconstructed from the computed centroid positions of each spot. The centroiding operator is linear, like the wave-front reconstruction problem. Consequently, the wave front estimated from a long-exposure image should be equal (apart from the error propagated during the reconstruction) to the average of many instantaneous wave fronts. This means that a Shack–Hartmann sensor gives us access to the average double-pass wave front $W_{dp}$. Therefore it is of interest to explore only the relationship between the single-pass wave front $W_1$ and the average double-pass wave front $\langle W_{dp} \rangle$.
Unfortunately, it is not possible to obtain an analytical expression for $\langle W_{th} \rangle$ from Eq. (3). Hence it was decided to make use of a model of retinal scattering similar to the one proposed by Marcos et al.\textsuperscript{6} for numerical simulations.

3. SIMULATION OF RETINAL SCATTERING

The retinal model of Ref. 6 treats scattering from the layer of cones as a far-field diffraction problem. Cones are treated as cylinders inserted into a medium of a given refraction index and are supposed to be approximately equally spaced and with random length variations. Light emerging back from each cone has a different phase from that of the light from adjacent cones. This is similar to the problem of light reflected by a rough surface. The total field at the pupil plane is the coherent superposition of the propagated field from each cone. Consequently, this is the Fourier transform of the field just after the layer of cones ($R$ from the model presented in this paper).

The model of Marcos et al. uses a retinal patch of 0.68°. The authors took experimental data of cone distributions and digitally manipulated them to correspond with retinal eccentricities from the central fovea to the near parafovea, with different mean values of cone spacing, ranging from 2.25 to 7 $\mu$m. It was assumed that the mean spacing between cones does not vary on the retinal patch under analysis. Each cone was assumed to be circular with a radius of 40% of the intercone spacing. The phase variation in the light emerging from the cones was assumed to have Gaussian statistics with zero mean and standard deviations varying from 0.049 to 0.27 $\mu$m.

The retinal patch is assumed to be illuminated by a Gaussian beam; hence the amplitude reflectivity of the surface is the cone mosaic described above multiplied by a Gaussian function, and the phase map is the optical path-length distribution of the cone matrix multiplied by $2\pi/\lambda$. The distribution at the pupil plane is simulated for a short exposure, and then 50 images are averaged. The cone distribution is supposed to be the same, and only the phase distribution is changed from one exposure to the next. The intensity of the scattered field obtained from these simulations is then fitted to a Gaussian of the form $10^{-p^2}$. This average corresponds with $\langle |R|^2 \rangle$ of the model presented in this paper.

The main difference between their model and ours is that for simplicity we used a one-dimensional model and, instead of having cones spaced in a pseudorandom fashion, we arranged them in a perfectly periodic array. The results obtained for $\langle |R|^2 \rangle$ were similar to those reported by Marcos et al.\textsuperscript{6} and in agreement with scattering theory.\textsuperscript{7–9} The retinal mosaic was constructed in the following manner. An empty vector of 2048 pixels was generated, corresponding to 200 $\mu$m of the retinal plane. The distance between cones was then decided, and the corresponding value $P$ in pixels was calculated. The empty vector was then converted into a Dirac comb of period $P$:

$$ p \quad p \quad p \quad \left[100 \cdot 0100 \cdots 0100 \cdots 0 \cdots \right].$$

If the length of the resulting vector was larger than 2048, the last incomplete period was removed, leaving only an integer number of periods in the vector. A single cone was defined as a square function of ratio width to period of 0.8 and multiplied by a Gaussian amplitude, as shown in Fig. 3. By convolving the Dirac comb with a single cone, we produced a perfectly periodic array of cones.

To simulate the phase variations introduced by each cone, we multiplied the Dirac comb by a random phase of unit amplitude $\exp(ir)$, where $r$ is a uniformly distributed random number between 0 and $2\pi$. Marcos et al.\textsuperscript{6} have shown that the diffused component of the scattered field dominates in the retinal case, and, consequently, phase variations between 0 and $2\pi$ are appropriate. After random phase changes are introduced in the Dirac comb, the convolution operation then generates an array of cones, all with the same amplitude but with different phase. The phase over each cone is constant. This vector corresponds to $R$ from Eq. (2).

The Fourier transform of this retinal mosaic is now computed in order to generate the instantaneous scattered field and its square modulus for visualizing the intensity distribution over the pupil plane. The wavelength was assumed to be 635 nm in order to compare the results generated by this model with wave-front data obtained by using light of the same wavelength.\textsuperscript{10} Up to this stage, the information relative to the entrance and exit pupils (including the propagating optical field) had not been used. The simulated field corresponded only to $R$.

As a way of checking the results of the one-dimensional model against those generated by the two-dimensional model of Marcos et al. and the predictions from scattering theory, we analyzed the average of the intensity of the scattered field of our model. This analysis was done for retinal mosaics with cones spaced between 1 and 4.4 $\mu$m. For each different cone spacing, 100 different mosaics with different random phases were created. For each mosaic, the intensity $\langle |R|^2 \rangle$ of the scattered field was calculated. At the end, the average $\langle \langle |R|^2 \rangle \rangle$ was computed and fitted to a Gaussian of the form $10^{-p^2}$. The expected values of $p$ were estimated in a similar fashion to that described by Marcos et al.\textsuperscript{6} from the Kirchhoff approxima-
pupil functions were generated over a pupil of radius 0.4 mm: one “even” and one “odd.” The phase of them is given by \( W_{\text{even}}(\xi) = \frac{\xi^2}{d^2} \) (defocus) and \( W_{\text{odd}}(\xi) = \frac{\xi^2}{d^2} - \frac{\xi}{d} \) (coma). The optical fields in the pupil plane are given by

\[
\mathcal{L}_1(\xi) = \exp\left[-i\frac{2\pi}{\lambda} W_{\text{even}}(\xi)\right],
\]

where \( W_{\text{even}} \) is either \( W_e \) or \( W_o \). This field is then convolved with the instantaneous scattered field obtained from the numerical simulations described in Section 3, and the result is then multiplied by \( \mathcal{L}_2(\xi) \) (cf. Eq. (3)). The phase is extracted from the resulting field and then divided by \( 2\pi\lambda \). Figure 2 shows the phase extracted from one of those simulations by using the defocused wave front and a retinal mosaic with a spacing of 1.5 \( \mu \)m between cones. Note that in a symmetric double-pass process we have \( \mathcal{L}_2(\xi_2) = \mathcal{L}_1(\xi_2) \).

This process was then repeated 100 times, and the average wave front was calculated. Figure 5 shows some of the results obtained. In all cases, the solid curve is the averaged double-pass wave front \( W_{4p} \), the dashed curve is the single-pass wave front \( W_{s0} \), and the dotted curve is the average wave front of the scattered field. In the first column, Figs. 5(a), 5(c), and 5(e) show the effects of the double-pass process on the even wave front \( W_e \), whereas the odd wave front \( W_o \) is partially kept (close to 0.3 \( \mu \)m of defocus), in the case, most of the phase information is lost after scattering. Figures 5(c) and 5(d) show a different behavior; whereas for the even case the phase information of the first pass is partially kept, in the odd case most of the phase information is lost after scattering, producing a double-pass average wave front very similar to that of a single pass. Finally, Figs. 5(e) and 5(f) show the case for larger cones (spaced 5 \( \mu \)m). In the even case, the scattering process keeps most of the phase information, producing a double-pass wave front almost twice the value of the single-pass one. In the odd case, the final double-pass wave front is smaller than the single-pass one, but without having been completely canceled out. Note that in a double-pass process without scattering, even aberrations duplicate while odd aberrations cancel out.

These numerical results are consistent with previous results reported by the authors. In this reference, single-pass versus double-pass measurements of ten ocular wave fronts were compared. The measurements were taken with a Shack–Hartmann sensor. It was reported that, within the accuracy of the system used, the experimental results did not show differences between single- and double-pass measurements or between symmetric and asymmetric double-pass measurements. None of the observed changes could be attributed to double-pass effects. The observed differences can be due to eye movements or changes in alignment of the eye with respect to the system between measurements.

All the measurements were taken at the fovea, where the cone spacing is the smallest of the retina and where cone diameters can be as small as 1.5 \( \mu \)m. The predictions of this model, however, suggest that if the cone mosaic is the main contributor to retinal scattering, double-pass effects could be an issue for larger eccentricities. However, other structures such as rods and capillary vessels and other retinal layers may enhance the scattering process, reducing the information bias produced by a double-pass process. Additionally, it is important to emphasize that the model of retinal scattering used here fails for eccentricities larger than 2°. This may be due again to retinal structures other than the cones.

The comparisons of Ref. 10 were taken by using three different configurations:

1. Single-pass measurements using lipofuscin fluorescence as described in Ref. 13.
3. Symmetric and asymmetric double-pass measure-
ments using circularly polarized light and collecting light that did not change its polarization state.

In the first group of experiments, lipofuscin autofluorescence was used to generate a source of light for the second pass completely decorrelated with the light from the first pass. The use of fluorescent light creates a completely incoherent source, producing light evenly distributed over the pupil plane. Contrary to the case of scattering light, the illumination is uniform and speckle free. Additionally, the phase of the fluorescent field is completely randomized and with zero mean. Lipofuscin accumulates at the retinal pigment epithelium behind the layer of cones.

543-nm light was used to excite retinal autofluorescence. At the exciting wavelength used, lipofuscin has a broad spectrum with a maximum approximately at 630 nm. The wavelength used for double-pass measurements was 635 nm. The small difference in wavelengths was not expected to introduce significant chromatic aberration.

The use of different polarization states attempted to discern between different components of the scattered fields. The intention in using linearly polarized light was to collect only depolarized light, that is, scattered light from the retina. The use of circularly polarized light was intended to collect only nondepolarized light. However,

Fig. 5. Average double-pass wave front after scattering. In each plot, the dashed curve is the single-pass wave front, the solid curve is the double-pass wave front, and the dotted curve is the phase of the scattered field. (a) Defocus. Cone spacing: 1 μm. (b) Coma. Cone spacing: 1 μm. (c) Defocus. Cone spacing: 2 μm. (d) Coma. Cone spacing: 2 μm. (e) Defocus. Cone spacing: 5 μm. (f) Coma. Cone spacing: 5 μm.
this was not entirely possible on account of the birefringent properties of the cornea\textsuperscript{15} and the retina.\textsuperscript{16} At the time when the experiments were performed, it was not possible to obtain careful measurements of the fast axis of the cornea. Consequently, the measurements obtained from both sets of experiments presented a combination of polarization states. Note, however, that the model presented in this paper is a scalar model and does not take into account polarization effects. But evidence suggests that scattered light from the retina preserves its state of polarization.\textsuperscript{17} This is in agreement with a scattering model where multiple scattering is not present, such as the one utilized here.

5. DISCUSSION

It is important to give now a physical insight of the process at the retinal plane. This can be done by analyzing Eq. (2). The interaction between the amplitude impulse response $A$ of the system with the retina determines the loss of information. In the model described above, the phase of the first pass is delayed differently by each cone, but the phase within each cone is kept unaltered. In general, for any scatterer this would be expected to be true within the area defined by the correlation length of the surface.

Suppose for a moment that the amplitude impulse response of the eye could be improved to its diffraction limit. The width of the amplitude impulse response is minimized in this case. If the diffraction-limited spot is as small as the diameter of a single cone, then all the phase information of the first-pass wave front is kept after being captured and re-emitted by a single cone (assuming that each cone behaves like a single-mode optical fiber). This situation is depicted in Fig. 6(a). On the other hand, if the amplitude impulse response of the eye is much larger than a single cone, the phase is distributed among many cones, producing a randomized phase, as shown in Fig. 6(b).

In the far field, that is, in the pupil plane, the optical field is the coherent superposition of the fields at the retinal plane. If the field is re-emitted by a few cones only, the far field will have fewer random variations than if it is re-emitted by many cones. The larger the number of random variations is, the closer the average wave front will be to zero. On the contrary, a small number of random variations over the field will tend to preserve the shape of the average scattered wave front.

In this context, Fig. 5 shows noticeably different behavior of even and odd aberrations; for example, in cases (e) and (f), even aberrations almost duplicate while odd aberrations are smaller but far from cancel out. This can be explained by using the same ideas exposed above. An optical field rich in higher frequencies will produce a broader PSF than that of a field poorer in higher frequencies. The two situations depicted in Fig. 6 are helpful to explain the behavior observed in Fig. 5. The case of defocus only (even aberration) produces a smaller PSF than that of the comatic field (odd aberration). Consequently, the even field suffers from smaller randomization, and the double-pass effects are more evident in this case.

These results are encouraging for the use of double-pass methods such as the Shack–Hartmann sensor for measuring the ocular aberrations, even in symmetric configurations. However, the authors are interested in measuring the wave-front aberrations of the eye for adaptive optical compensation for high-resolution retinal imaging. Under these circumstances, the scatterer (for instance, retinal cones) becomes resolvable by the optics of the system. As shown in Fig. 6(a), the amplitude response of the eye may become comparable with the size of a single scatterer, enhancing the effects of the double-pass process. Further experimental work in this direction is still required. In the context of adaptive optics, it may be that an error in the measurement of the wave-front aberration simply means that a larger number of iterations are required in the adaptive optics control loop, thus reducing the bandwidth of the system. In the context of wave-front sensing, the current wave-front sensing techniques are probably not accurate enough for measuring the small differences that can be introduced by the double-pass process in the ocular wave front.

One last issue has to be raised.\textsuperscript{3} Artal \textit{et al.} show that there is a loss of information in the presence of a rough scatterer. This is apparently in contradiction with the results of this paper; however, the reader has to be aware that the measured quantities are different. The double-pass average PSF is proportional to the average of the square modulus of the Fourier transform of Eq. (3):
where $R$ is the Fourier transform of $\mathcal{R}$ and $D$ is a proportionality constant. If $R$ is $\delta$-correlated, that is, if it satisfies the condition

$$\langle R(-x_2, -y_2; t) R^{*}(x'_2, -y'_2; t) \rangle = \delta(x'_2 - x_2, y'_2 - y_2),$$

where the superscript asterisk denotes complex conjugate, then, in this case,

$$I_{dp}(x_2, y_2; t) = D'[A_1(-x_2, -y_2)]^2 \otimes [A_2(x_2, y_2)]^2 = D'[I_1(-x_2, -y_2) \otimes I_2(x_2, y_2)]$$

$$= D' I_1(x_2, y_2) \otimes I_2(x_2, y_2),$$

where $D'$ is a proportionality constant not necessarily equal to $D$. $I_i(x_j, y_j)$ is the PSF of the $i$th pass evaluated in the $i$th pupil plane, and $\otimes$ denotes convolution. This change of operation is due to the relation between correlation and convolution for real numbers. This expression was derived for the first time by Artal et al.\textsuperscript{5} Experimental results reported by Navarro and Losada\textsuperscript{18} and by Artal et al.\textsuperscript{19} are in agreement with a retinal structure with a correlation length much smaller than the ocular PSF (that is, approximately $\delta$-correlated) at the fovea. The double-pass PSF is the result of an incoherent imaging process. The moving scatterer produces, after a long exposure time, an incoherent extended object. It is this extended object that is imaged by the optics of the eye in the second pass, as shown in Fig. 7. The phase information of the first pass is lost after interaction with the scatterer; however, it leaves a mark in the shape of the first-pass PSF, and this mark becomes evident, for instance, in the double-pass PSF. Instead, the phase of the double-pass field does not contain this information; it is contained only in the amplitude of the double-pass field and consequently is detectable only when dealing with the intensity of the optical field.

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