Double lateral shearing interferometer for the quantitative measurement of tear film topography

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1. Introduction

The evaluation of the optical quality of the human eye beyond spectacle prescription (or wave-front sensing) has recently become an active field, given the number and importance of the applications that can benefit from it. The most widespread of these is refractive surgery in its different techniques: photorefractive keratectomy (PRK), laser-assisted in situ keratomileusis (LASIK), and custom ablation.\textsuperscript{1,2} Wave-front sensing in the eye can also—through deconvolution or adaptive optics—improve the performance of retinal imaging instruments such as fundus cameras\textsuperscript{3–7} or confocal laser scanning ophthalmoscopes.\textsuperscript{8} A number of wave-front sensors have been tested in the eye, showing intersubject variability in the estimated root mean square (rms) of the wave-front aberration of the order of 0.1 \(\mu\)m for pupils over 5 mm in diameter.\textsuperscript{9–17} These values are not an issue for spectacle prescription (0.1 \(\mu\)m of defocus for a 4-mm pupil corresponds to 1/5 diopter, which is around the minimum step between consecutive ophthalmic lenses), but they are of concern when considering a diffraction-limited performance imaging system for the eye because the wave-front rms should be kept below \(\lambda/14\) (Marechal’s criterion), that is, 0.03–0.05 \(\mu\)m for visible wavelengths. The understanding of the sources of this variability in ophthalmic wave-front sensing is crucial to the improvement of refractive surgery and high-resolution retinal imaging instruments.

It is likely that the largest source of variability of the optical quality of the eye is the fluctuation of the accommodation,\textsuperscript{18,19} which depends on several factors such as pupil diameter, target luminance, form, vergence, and contrast, and which can have amplitudes as large as 0.5 diopters. For this reason, it is common practice in research environments to paralyze or reduce these fluctuations by use of drugs (usually cyclopentolate).\textsuperscript{12,15,20–22} A less significant source of variability seems to be the axial length change of the eye when the heart beats.\textsuperscript{23} The role of eye movement in wave-front sensing measurements has, to our knowledge, not yet been studied on its own, although the movements of the eye are well understood.\textsuperscript{24} There seems to be agreement in the ophthalmic wave-front sensing community about the tear film playing a role in the variability of measurements,\textsuperscript{6,14–16,20,22,25–30} although to our knowledge there is no quantitative study to date. Most of the research related to this topic so far has been done for the extreme situation of tear breakup, like that by Tutt \textit{et al.}\textsuperscript{22} showing that the tear breakup affects the contrast of retinal images and the spot pattern in a Shack–Hartmann sensor. Some efforts by Himebaugh \textit{et al.}\textsuperscript{28} to provide a technique to study the changes on the tear topography made in parallel to and independently of the research described in this paper led to a retroillumination instrument, which is yet to produce definite results. A more suitable technique to study the tear topography was proposed by Licznerski \textit{et al.},\textsuperscript{31} where they obtained information about the tear topography using lateral shearing interferometry.
However, this experimental setup had an important limitation in that the tear topography estimation was made with a gradient projection along only one direction.

In this paper we present an improved version of Licznerski et al.’s interferometer, designed for quantitative analysis of tear topography dynamics and its effects of visual performance. The instrument estimates the tear topography with two glass wedges to produce pairs of lateral shearing interferograms. We give a brief description of the theory of the interferometric technique, followed by a description of the instrument and data processing. Data collection and analysis are presented, as are the different features found in tear topography; their effect on the topography estimation are also discussed.

2. Theory

In the lateral shearing interferometer described here, the information of the tear topography $t$ is carried by the phase $\phi$ of a beam reflected on the front surface of the precorneal tear film. These two functions are linked by

$$\phi = \frac{2\pi}{\lambda} (2t), \quad (1)$$

where $\lambda$ is the wavelength of the light used in the experiment and the factor of 2 arises from reflection at normal incidence. The beam reflected by the tear, containing the phase information to be measured, is split into two beams laterally shifted, which are recombined to produce an interferogram. The lack of a static coherent reference beam makes the resulting interferogram less sensitive with respect to eye movement. If we assume that the amplitude division produces two beams with equal polarization states, and equal amplitude and phase profiles, then the equation that describes the intensity pattern resulting from the interference of the sheared copies of the electric field $E_\phi(r)$ and $E_\phi(r+s)$ is

$$I(r) = I_\phi(r) + I_\phi(r+s) + 2[I_\phi(r)I_\phi(r+s)]^{1/2}\cos[\phi(r+s) - \phi(r)], \quad (2)$$

where $r$ is the position vector in the interferogram plane, $s$ is the shear between the two beams, $I_\phi(r) = |E_\phi(r)|^2$ and $\phi$ are the phase of the complex electric field $E(r)$, the function of interest. It is implicitly assumed that the spatial and temporal coherence of the beams are such that the modulus of the complex degree of coherence$^{32}$ is virtually one for the shear and wave front under consideration. To reconstruct the two-dimensional phase map $\phi$, at least two interferograms with different shear directions (not necessarily orthogonal) are needed because the phase difference in Eq. (2) is insensitive to phase functions constant along the direction of shear.

We extracted the phase information from the intensity patterns by spatially modulating the intensity of the interferograms, introducing a controlled amount of tilt between the two interfering beams and using the phase recovery algorithm proposed by Takeda et al.$^{33}$ In this way, a phase map can be calculated from a single pair of recorded interferograms (with shear in different directions), as opposed to multiple interferograms required in phase-stepping interferometers. The spatial frequency of the intensity modulation $f_c$ acts as a phase carrier frequency, modifying Eq. (2) to

$$I(r) = I_\phi(r) + I_\phi(r+s) + 2[I_\phi(r)I_\phi(r+s)]^{1/2}\cos[\phi(r+s) - \phi(r)] - 2\pi f_c \cdot r. \quad (3)$$

If the modulus of the highest spatial frequency of the tear topography is smaller than half of the modulus of the modulation frequency, then the Fourier transform of the interferogram will consist of three well-separated peaks. The central peak (dc term) corresponds to the first two terms on the right-hand side of Eq. (3), and the two side peaks (ac terms) are associated with the third term, modulated at the carrier frequency $f_c$. By retaining only one of these side peaks, displacing it to the origin of frequency coordinates, and inverse Fourier transforming, we obtain the complex quantity $I_{\text{filtered}}$ given by

$$I(r)_{\text{filtered}} = 2[I_\phi(r)I_\phi(r+s)]^{1/2}\exp \{\pm i[\phi(r+s) - \phi(r)]\}, \quad (4)$$

where the sign on the exponential depends on which side peak is retained. Then, by taking the complex logarithm of $I_{\text{filtered}}$ and taking the real and imaginary parts, one obtains the intensity and wrapped $[(-\pi, \pi)]$ phase maps of the interferograms, respectively.$^{33}$

3. Instrument Description

A. Illumination Branch

The purpose of the illumination branch of the interferometer, shown in Fig. 1, is to produce a smooth (ideally uniform) normal illumination over a circular region of the front surface of a subject’s precorneal tear film.

The light source used was a linearly polarized single-mode He–Ne laser with an output of 2.5 mW at 632.8 nm, followed by an absorption neutral-density filter to keep the radiation reaching the eye around 5 μW, which is 3000 times below safety limits for exposures of up to 100 s according to the British and European standard for safety of laser products (BS EN 60825-1:1994 with amendments 1, 2, and 3). The output of the laser is magnified and spatially filtered with a telescope formed by a $\times 40$ microscope objective with a numerical aperture of 0.65, a 10-μm pinhole at the focal plane of the microscope objective, and a 250-mm focal-length collimating lens with an aperture stop of 15 mm in diameter. The resulting collimated beam is then directed toward the eye with a
laser-line coated polarizing beam splitter and a $\lambda/4$ wave plate with the axis at 45° with respect to the polarizing beam splitter. The combination of these two elements maximizes the light reflected back from the eye toward the imaging branch of the interferometer and reduces the reflections from the beam splitter itself in comparison with a nonpolarizing beam splitter by 2 orders of magnitude.

The last element of the illumination branch, which is shared with the imaging branch, is a 40-mm focal-length achromat that focuses the illumination beam to the center of curvature of the cornea, making the illumination normal with respect to the tear surface. The focal length and diameter of the lens were chosen to keep the eye clearance around 15 mm for comfort (which is the usual distance for spectators) and to maximize the size of the illuminated area while keeping costs and aberrations low. The full area of the lens was not used so as to allow some tolerance for eye movement. For a typical cornea of radius 8 mm, the diameter of the illuminated area of the tear was approximately 3.4 mm.

B. Imaging Branches

The first part of the two imaging branches consists of the achromat shared with the illumination branch and a 5:1 afocal reducer system, which combined produce an image of the tear surface with unit magnification. That image is then copied onto a camera by two 1:14$^f$ systems folded in a three-dimensional arrangement and with the glass wedges inserted at 45° in the system as indicated in Fig. 1. In this way, each glass wedge produces a pair of horizontally (vertically) sheared and tilted copies of the incident wave front when the impinging light is reflected on the front and back surfaces with almost equal intensity, thus producing interferograms with high contrast. The influence of the wedges in transmission was neglected.

Using paraxial optics it can be shown that the amplitude of the interferograms’ shear depends linearly on the position of the wedges along the optical axis of the interferometer. Thus we can tune the shear amplitudes to a desired value simply by displacing the wedge along the optical axis of the interferometer, providing some tolerance to the wedge manufacture and more importantly some adjustment of the signal-to-noise ratio on the phase estimation from the interferograms.

When observing the interferograms produced by the experimental setup described above, one can observe a clear difference in their intensities because of the dependence of the reflection coefficients of the glass wedges on the incident state of polarization. Rotation of a $\lambda/2$ wave plate inserted in the optical system just before the first wedge allowed the interferograms’ intensities to be equalized while the interferogram contrast was kept higher than 0.99.

C. Wedges

The tilt between the reflections produced by the wedges should be large enough so that the spectra of the ac and dc terms of the interferogram intensity do not overlap in the Fourier domain. On the other hand, it should be small enough for the resulting fringes in the interferogram to be adequately sampled by the camera being used. Therefore the choice of tilt should be a compromise between these two factors. Because we have no prior information on the spatial-frequency content of the tear topography, we chose the tilt between the interfering wave fronts so that the carrier frequency of the interferogram could be adequately sampled (six to eight camera pixels per fringe).

Once the tilt and shear to be used in the experimental setup were defined, we calculated the wedge angle and thickness by inverting and combining Eqs. (A1), (A2), and (A5) (see Appendix A). The two uncoated BK7 glass wedges used for the interferometer described here had angles of 11 and 13 arc min and mean thicknesses of 0.35 and 0.70 mm, respectively. The beam width at the wedges was 3 mm, and the shear variation across the beam was kept below 2%.

4. Data Processing

The processing of the pairs of raw interferograms to obtain the tear topography maps consists of four stages: image registration, phase recovery, phase unwrapping, and integration. In the image registration stage each interferogram is aligned with respect to its pair. In the phase recovery we can extract the phase difference maps $\phi(r+s) - \phi(r)$ from the interferograms using Takeda et al.’s asymmetric filtering in the Fourier domain. For the phase unwrapping a least-squares unwrapping algorithm was used. It can be shown that the estimation of an unwrapped phase map $\psi_u$ from a wrapped phase map $\psi_w$ over a
grid of points \((i, j)\) can be formulated as the minimization of the error \(e^2\) defined as

\[
e^2 = \sum_{i,j} \left( \left( \Delta \psi_u(i, j) - \Delta \psi_v(i, j) \right)^2 + \left( \Delta \psi_v(i, j) - \Delta \psi_u(i, j) \right)^2 \right),
\]

where \(\Delta \psi_u(i, j) = \psi(i, j) - \psi(i-1, j)\) and \(\Delta \psi_v(i, j) = \psi(i, j) - \psi(i, j-1)\). Then the problem becomes that of solving an overdetermined set of equations, which, because of its large size, we solved using a least-squares fast-Fourier-transform-based method proposed by Poyneer et al.\(^{25}\) Finally, we achieved the calculation of the tear topography maps, or equivalently \(\phi\), by a least-squares fitting of the pairs of unwrapped phase maps \(\psi_u^s\) and \(\psi_v^s\) by minimizing the error

\[
\delta^2 = \sum_{i,j} \left[ \left( \phi(i + s_x, j) - \phi(i, j) - \psi_u^s(i, j) \right)^2 + \left( \phi(i, j + s_y) - \phi(i, j) - \psi_v^s(i, j) \right)^2 \right],
\]

where \(s_x\) and \(s_y\) are the interferogram shear amplitudes in pixel units. This problem, similar to the phase unwrapping, was solved with a least-squares fast-Fourier-transform-based method that is a generalization of that proposed by Poyneer et al. for arbitrary integer values of \(s_x\) and \(s_y\).\(^{36}\)

In some tests in which a glass sphere is used instead of a real eye showed that the repeatability of the r.m.s. of the topography measurements was around \(\lambda/60\). In Appendix B we calculate how the effect of undesired reflections propagate through the phase recovery algorithm; we found that in our experimental setup the associated error is less than \(\lambda/40\), which given the measured phase maps is negligible in comparison with the other sources of error (note that this is an error in the interferogram phase map, not in the topography). The errors introduced by the automated interferogram registration were estimated, through simulations, to be around 13%.\(^{37}\) Finally, the error due to the data that do not comply with the assumptions on which the data processing is based (such as correctly sampled band-limited spatial spectra) was found to be around 6%.\(^{37}\) If we then take a conservative approach, we can estimate the total uncertainty in the tear topography r.m.s. \(\sigma_{\text{RMS}}\) by combining the errors assuming that their sources are uncorrelated, obtaining \(\sigma_{\text{RMS}} \approx (\lambda/60)^2 + (14\%)^2)^{1/2}\).

5. Data Collection Protocol

The volunteers that participated in the experiments leading to this study were informed of the experimental protocol, approved by the St. Mary’s Local Research Ethics Committee of the Kensington, Chelsea, and Westminster Health Authority. All the information was coded and strictly confidential.

A bite registration with soft dental wax was made and mounted on a \(x-y-z\) translation stage for precise positioning of the subject with respect to the instrument. When the subject was aligned with respect to the optical system, he or she was asked to remain as steady as possible and to blink normally while the data were recorded.

6. Results

To illustrate the performance of the instrument and data processing, and more importantly their limits, we discuss eight different tear topography cases shown in Figs. 2 and 3. These interferograms were chosen to illustrate the different features found in 5000 recorded pairs of interferograms. The smooth tear surface corresponding to the interferogram in Fig. 2(a) is by far the most representative situation with over 66% of the recorded data. The postblink tear undulation in Fig. 2(e), the bubbles shown in Fig. 2(i), the eyelid-produced bumps and ridges illustrated in Fig. 2(m), and tear breakup in Fig. 3(e) are much less frequent features. The extreme undulation of tear surface shown by the interferogram in Fig. 3(a) is very rare (less than 1%), and the tear topography roughness in Figs. 3(i) and 3(m) was found only in precontact lens tear surface (both soft and hard).

The second columns of Figs. 2 and 3 show the frontal view of the three-dimensional plot of the spectra of the corresponding interferograms. The region of the spectra within the dashed vertical lines indicates the area where the dc term is to be found. It can be seen that for smoother tear topographies (Figs. 2(b), 2(f), 2(j), 2(m), and 2(f)) there is virtually no overlapping between the dc and ac terms, and thus quantiative data processing as described in Section 4 will be possible. However, when the tear front surface becomes rougher or undulated as shown in Figs. 3(b), 3(j), and 3(n), the overlapping of the dc and ac term indicates that quantitative analysis in the way described above is not possible.

It is interesting to observe the intensity and wrapped phase maps resulting from the interferogram phase recovery in the two rightmost columns of Figs. 2 and 3. There are two points to note from these columns: the nonuniformity of the intensity profiles and the singularities in the phase maps. The changes in the intensity maps are due to the dependence of the tear reflection coefficient with the angle of incidence in steep tear topography features (e.g., bubbles). In more extreme cases, when the steepness of tear features is such that the light reflected by the tear is not collected by the imaging branches of the interferometer (vignetting), the intensity profile takes a zero value and consequently the phase at that point is undefined. These points with an undefined phase can be identified in Figs. 2 and 3 at the ends of the \(2\pi\) phase line discontinuities other than those at the pupil edges. It should also be noted that high spatial-frequency content on tear topography, such as that shown in the first, third, and fourth rows of Fig. 3, produces severe aliasing of the ac and dc terms (second column); thus the intensity and phase profiles are not useful for quantitative analysis.

Once the phase maps are unwrapped, then the tear topography can be calculated; by dividing by the mean refractive index of the tear (1.337), we can cal-
culate the wave-front aberration maps. Figure 4 shows four wave fronts, each reconstructed by use of the vertical, the horizontal, and both lateral shearing interferograms, illustrating the need for both interferograms for a good estimation of the tear topography. The spacing between contours is $\lambda/14$ with $\lambda = 632.8$ nm, and the number in the bottom left corner of each map is the wave-front rms.

An important point to note about the magnitude of the aberrations from each individual map shown in Fig. 4 is that the sum of the aberrations of the tear, optical system, and corneal surface is shown. When studying the topography dynamics, the corneal aberrations and those aberrations from the instrument itself can be removed simply by means of subtracting one of the maps from the rest of the series being dealt with.

7. Conclusions
We have presented a double shearing interferometer to evaluate the precorneal tear film topography quantitatively, and its performance on different tear topography features was discussed. The interferometer successfully estimated tear topography maps, with errors of the order of 15% and a minimum error of $\lambda/60$, therefore proving that the interferometer can be used to study the effect of tear topography in the optical quality of the eye.
We note that the front surface of the tear in front of contact lenses is rough; and although a quantitative analysis of the topography was not possible when the typical roughness was present, we believe the subject should be studied further given the high number of contact lens users.

Appendix A: Glass Wedge Design Formulas

The amplitude division for the interferometer is achieved by use of the reflections from the front and back surface of uncoated wedged glass plates at 45° of incidence with respect to the optical axis of the interferometer (multiple reflections are neglected). Figure 5 shows the geometry and angle nomenclature used for the choice of wedge parameters. From use of Snell's and the reflection laws it can be shown that, for a ray incident on the wedge at the point $A$, the tilt $t = |\alpha_5 - \alpha_1|$, between the reflection on the front and back surface is given by

$$t = \left| \arcsin \left( n_w \sin \left( 2\delta + \arcsin \left( \frac{\sin \alpha_1}{n_w} \right) \right) \right) - \alpha_1 \right|,$$

where $n_w$ is the relative refractive index of the wedge in air. Equation (A1) shows that the tilt between each pair of rays resulting from the same incident ray depends on the angle of incidence $\alpha_1$. Therefore, unless all the rays of the incident beam at the wedge are parallel, the reflected wave fronts will be dis-

Fig. 3. As in Fig. 2. From top to bottom: undulated tear surface due to blink prevention, tear breakup, and two extremely rough precontact lens tear surfaces.
torted. Hence the wedges must be placed in a region of the optical system where the incident beam is collimated.

If we now calculate the shear \( s = \overline{AC} \) introduced by the wedge using trigonometry, we have for \( \delta \ll 1 \) that

\[
s = \sin \left[ 2 \arcsin \left( \frac{\sin \alpha_1}{n_w} \right) \right] \cos^{-2} \left[ \arcsin \left( \frac{\sin \alpha_1}{n_w} \right) \right] \frac{\delta AO}{n_w},
\]

where it can be seen that the shear is nonuniform across the wedge, and thus the wave front reflected on the back surface of the wedge will be nonuniformly stretched in the direction of the shear. The relative change in shear \( D_s \) across the beam width \( w \) at the wedge can be defined as

\[
D_s = 2 \frac{s_{\text{max}} - s_{\text{min}}}{s_{\text{max}} + s_{\text{min}}},
\]

where \( s_{\text{min}} \) and \( s_{\text{max}} \) are the shear of the rays incident on the wedge closest and furthest from the wedge apex, respectively. By using approximation (A2) assuming a collimated incident beam, we can rewrite the relative change in shear across the beam as

\[
D_s = \frac{w}{AO_{\text{mean}}},
\]

where \( w \) is the beam width at the wedge. This suggests that the shear nonuniformity can be kept below a given tolerance \( T \) if we make the distance \( AO_{\text{mean}} \)
large in comparison to the beam width or equivalently make the wedged plate thicker:

\[ T > \frac{\omega}{AO_{\text{mean}}} \]  \hspace{1cm} (A5)

Appendix B: Effect of Undesired Reflections in Takeda’s Phase Recovery Method

A source of noise when Takeda et al.’s phase recovery method is used (see Section 2) is the unwanted reflections from optical elements and, in this case, from surfaces within the eye. We now estimate a bound for the error that these reflections can introduce, knowing their intensity relative to the main signal (reflection from the tear front surface).

We can describe the electric field at the output of the tear film experiment as the superposition of two sheared and tilted copies of the electric field to be studied \( E_T \) and the undesired reflections \( E_R \). The observed intensity \( I = |E|^2 \) at the output of the interferometer will then be

\[
I(\mathbf{r}) = |E_T(\mathbf{r}) + E_T(\mathbf{r} + \mathbf{s})\exp(2\pi i \mathbf{f} \cdot \mathbf{s}) + E_R(\mathbf{r})
+ E_R(\mathbf{r} + \mathbf{s})\exp(2\pi i \mathbf{f} \cdot \mathbf{s})|^2, \tag{B1}
\]

where we assume that the source has a coherence length greater than the length of the optical system. If we now apply Takeda’s Fourier filtering algorithm, we obtain the recovered phase \( \phi_{\text{rec}} \):

\[
\phi_{\text{rec}}(\mathbf{r}) = \arctan \left( \frac{I_{T,T} \sin \phi_{T,T} + I_{R,R} \sin \phi_{R,R} + I_{T,R} \sin \phi_{T,R} + I_{R,T} \sin \phi_{R,T}}{I_{T,T} \cos \phi_{T,T} + I_{R,R} \cos \phi_{R,R} + I_{T,R} \cos \phi_{T,R} + I_{R,T} \cos \phi_{R,T}} \right), \tag{B2}
\]

where \( I_{b,\text{obs}}(\mathbf{r}) = [I_b(\mathbf{r})I_b(\mathbf{r} + \mathbf{s})]^{1/2} \), \( \phi_{b,\text{obs}}(\mathbf{r}) = \phi_b(\mathbf{r}) - \phi_b(\mathbf{r} + \mathbf{s}) \), and \( \phi_b \) is the phase of the electric field \( E_b \). If the amplitude of the undesired reflections is much smaller than the amplitude of the light reflected from the tear film, i.e., \( I_R/I_T \ll 1 \), and it is assumed that the intensity profiles are slowly varying over the pupil \( [I_a(\mathbf{r}) \approx I_a(\mathbf{r} + \mathbf{s})] \), then, to first order, we can neglect the terms in \( I_{R,R} \) and approximate the recovered phase by

\[
\phi_{\text{rec}}(\mathbf{r}) = \phi_{T,T}(\mathbf{r}) + (\sin \phi_{R,T}(\mathbf{r})
+ \sin \phi_{T,R}(\mathbf{r}) \cos \phi_{T,T}(\mathbf{r}) - \cos \phi_{R,T}(\mathbf{r})
+ \cos \phi_{T,R}(\mathbf{r}) \sin \phi_{T,T}(\mathbf{r})) \left[ \frac{I_{R,T}(\mathbf{r})}{I_{T,T}(\mathbf{r})} \right]. \tag{B3}
\]

If we now define the phase error \( \phi_{\text{error}} \) as the difference between the recovered phase and \( \phi_{T,T} \), it is possible to show that this error is bounded by

\[
\max(\left| \phi_{\text{error}} \right|) = 2 \left[ \frac{I_{R,R}(\mathbf{r})}{I_{T,T}(\mathbf{r})} \right]^{1/2}. \tag{B4}
\]

We can now use this approximate formula and the measured value for \( I_{R,R}/I_{T,T} \) for the instrument and the eye (1/200 and 1/150, respectively) to see that the undesired reflections produce a maximum phase error of approximately \( \lambda/40 \). It is important to remember that this error is not on the tear topography but on the phase difference maps. These errors are negligible in comparison with the other sources of noise and can therefore be neglected.

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