Chromatic effects of the atmosphere on astronomical adaptive optics

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The atmosphere introduces chromatic errors that may limit the performance of adaptive optics (AO) systems on large telescopes. Various aspects of this problem have been considered in the literature over the past two decades. It is necessary to revisit this problem in order to examine the effect on currently planned systems, including very high-order AO on current 8–10 m class telescopes and on future 30–42 m extremely large telescopes. We review the literature on chromatic effects and combine an analysis of all effects in one place. We examine implications for AO and point out some effects that should be taken into account in the design of future systems. In particular we show that attention should be paid to chromatic pupil shifts, which may arise in components such as atmospheric dispersion compensators. © 2008 Optical Society of America

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1. Introduction
The next generation of extremely large telescopes (ELTs) will employ adaptive optics (AO) to correct the wavefront aberrations introduced by atmospheric turbulence and hence provide for diffraction-limited imaging at unprecedented levels of angular resolution. Several ELT designs have been proposed having primary mirror diameters up to 100 m [1]. However, the largest aperture telescope being proposed at the moment is the European ELT (E-ELT), which would have a diameter of 42 m [2]. AO is considered a fundamental part of all ELT designs, and the great expense and effort required to build an ELT is frequently justified within the context of diffraction-limited imaging. A prime example is the search for exoplanets, which is considered a major scientific goal for these telescopes. It will only be possible to detect exoplanets if the wavefront correction is almost perfect, requiring the use of very high-order (or “extreme”) AO, a coronagraph, smooth optics, and precise calibration of residual errors [3,4]. Systems under consideration aim for Strehl ratios in excess of 0.9 at the imaging wavelength.

It is well known that atmospheric dispersion causes chromatic effects in astronomical imaging. The best known effect is the displacement toward the zenith in the apparent position of celestial bodies, which increases as they approach the horizon. This effect has been recognized since ancient times, and a correct explanation in terms of atmospheric refraction was given by Ptolemy in his Almagest [5]. Atmospheric refraction has other effects that may be important when it is required to correct wavefront errors with an AO system. These effects were considered in some detail in the early 1980s in a series of papers by Wallner [6,7]. It could be concluded from that work that the effects are negligible for what were then typical aperture sizes of telescopes with AO (1–4 m). It is of interest to revisit this area to verify that the effects of atmospheric refraction will also be negligible for the upcoming generation of ELTs. For completeness, we will also check the effect of atmospheric diffraction, which was previously considered by Hogge and Butts [8].

Chromatic errors may be of importance when wavefront correction is over a broad bandwidth, with
the wavefront sensing at the same mean wavelength as the correction, or when wavefront correction is at wavelengths significantly different from the wavelength of wavefront sensing. Broad bandwidth wavefront correction is especially necessary in the case of observing very faint objects while broad bandwidth wavefront sensing is required to improve sky coverage. An example of a system having very different wavelengths for wavefront sensing and correction is one using a sodium laser guide star at 0.589 μm for wavefront sensing and providing corrected images in the K band at 2.2 μm.

It is well known that seeing gets worse as a function of increasing zenith angle, due to the increasing path through turbulence [9]. The Fried parameter, \( r_0 \), which measures the spatial coherence of the wavefronts, decreases with the zenith angle, \( \xi \), as \( \sec^{3/2}(\xi) \). The performance of AO systems will likewise degrade as a function of the zenith angle, and chromatic effects (most of which depend on the zenith angle) will only be relevant if they significantly degrade performance, i.e., if the Strehl ratio is already small for large zenith angles, then a further reduction in the Strehl ratio due to chromatic effects is of little consequence. Assuming that AO performance is limited by a fitting error due to having a finite number of actuators, temporal bandwidth error, and time delay in the control response, then it can be shown that the Strehl ratio depends on \( r_0 \) according to [10]

\[
SR = \exp(-A r_0^{-5/3}),
\]

where \( A \) is a constant depending on the system. Combining this relation with the zenith dependence of the Fried parameter gives

\[
SR = \exp(-B \times \sec^{3/2}(\xi)),
\]

where \( B \) is also a constant. Figure 1 shows the zenith dependence of the Strehl ratio for two systems, one designed to have a Strehl ratio of 0.99 at zenith and the other to have a Strehl ratio of 0.95 at zenith. The fall-off in performance is seen to be quite slow when the system is designed to give a very high Strehl ratio at the zenith. This is because the fitting error remains small for such a high-order system even when observing at a high zenith angle.

2. Diffractive Effects

In the geometric optics approximation, a plane wave passing through a turbulent layer suffers random phase delays but does not change direction. The wave is also diffracted by the inhomogeneities in the refractive index, and the propagation of the wave will give rise to scintillation in the far field of the turbulent layer. This effect is well known [9] and accounts for a certain degradation of the performance that can be obtained with an AO system that only corrects phase. Since diffraction depends on wavelength, the scintillation will display some chromaticity. The effect of using a finite spectral bandpass when measuring the index of scintillation was considered by Tokovinin [11]. Hogg and Butts [8] examined the wavelength dependence of scintillation and present the following expression for the resulting phase error variance (in square radians) at \( \lambda_2 \) when the wavefront phase is measured at \( \lambda_1 \):

\[
s^2 = \frac{4.08}{\pi} k_0^2 \int_0^L C_z^2(z) \frac{d z}{d K} \int_0^{\infty} K^{-8/3} d K \\
\times \left\{ 1 - \left( \frac{4}{KD} \right)^2 \left[ J_1 \left( \frac{KD}{2} \right) \right]^2 \right\} \\
\times \left\{ \cos \left[ \frac{z K^2}{2 k_1} \right] - \cos \left[ \frac{z K^2}{2 k_2} \right] \right\}^2,
\]

where \( C_z^2(z) \) is the structure constant for the turbulence-induced refractive index fluctuations, \( K \) is the spatial frequency, \( D \) is the aperture diameter, \( k_1 \) and \( k_2 \) are the wavenumbers at \( \lambda_1 \) and \( \lambda_2 \), respectively, and \( L \) is the length of the propagation path. This result was derived using an expression for the phase structure function previously published by Fried [12]. When the observation is carried out at zenith angle \( \xi \), then \( z \) has to be replaced by \( z \sec \xi \) and \( d z \) replaced by \( d z \sec (\xi) \). Here we use this expression to predict the Strehl ratio over a range of correction wavelengths when the wavefront sensing is carried out at \( \lambda_1 = 0.589 \) μm.

A seven-layer model for turbulence is used. The model was derived by Femenía and Devaney [13] for the Observatorio del Roque de los Muchachos (ORM) on La Palma in the Canary Islands. It is based on data from six balloon flights [14]. The data from the balloon flights were normalized to have the same total Fried parameter \( r_0 \) as the ORM annual mean, \( r_0(0.5 \mu m) = 0.15 \) m, and a continuous average profile generated. The seven-layer profile was obtained by performing a discrete fit to the continuous average profile matching the first 14 moments of the continuous turbulence profile. The profile is presented in Table 1. We calculate the Strehl ratio from the wavefront variance using the “extended Méréchel approximation,” valid for Strehl ratios > ~0.2 [9]. Figure 2 shows the Strehl ratio as a function of wavelength,
Table 1. ORM Seven-Layer Profile of Femenía and Devaney [13]

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>$C_{2}^2 \lambda (m^{-1})^2$</th>
<th>$C_{7}^2 \lambda$</th>
<th>$r_{0}(0.5 \mu m)/(\lambda m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>$8.08 \times 10^{-14}$</td>
<td>23.0</td>
<td>0.363</td>
</tr>
<tr>
<td>463</td>
<td>$1.04 \times 10^{-13}$</td>
<td>29.6</td>
<td>0.312</td>
</tr>
<tr>
<td>1483</td>
<td>$8.64 \times 10^{-13}$</td>
<td>24.6</td>
<td>0.348</td>
</tr>
<tr>
<td>4840</td>
<td>$3.86 \times 10^{-14}$</td>
<td>11.0</td>
<td>0.565</td>
</tr>
<tr>
<td>11122</td>
<td>$2.25 \times 10^{-14}$</td>
<td>6.4</td>
<td>0.781</td>
</tr>
<tr>
<td>14806</td>
<td>$1.76 \times 10^{-14}$</td>
<td>5.0</td>
<td>0.905</td>
</tr>
<tr>
<td>18635</td>
<td>$1.40 \times 10^{-15}$</td>
<td>0.4</td>
<td>4.13</td>
</tr>
</tbody>
</table>

$\lambda_2$, with $\lambda_1 = 0.589 \mu m$ for a zenith angle of 45°. The effect is seen to be approximately 1% in the Strehl ratio when the science observation is in the red or near infrared. Wavefront correction shortward of the sodium wavelength is severely degraded; a high Strehl ratio at such short wavelengths is very difficult in any case. The effect of the aperture diameter is very small for apertures larger than a few centimeters.

3. Refractive Effects

The refractive index of air depends on wavelength, and this affects the process of imaging through the atmosphere in three ways. First, rays at different wavelengths are bent by different angles when passing through the atmosphere, and this gives rise to an angular spread in image size. Second, the optical path errors introduced by turbulence depend on wavelength, and this will give a chromatic error even when observing at zenith. Finally, differential bending of rays of different colors causes them to pass through different regions of the turbulence, giving rise to a chromatic anisoplanatism.

A. Angular Refraction

Astronomers are most familiar with the effects of atmospheric refraction on the apparent position of celestial objects. The Earth's atmosphere bends incoming light rays causing objects to appear higher above the horizon than would be the case in the absence of an atmosphere. Dispersion results in rays of different wavelengths, which were parallel at the top of the atmosphere entering the telescope aperture at different angles. When an image is formed, the rays will be brought into focus at slightly different positions in the focal plane, giving rise to a chromatic blurring, which is in the direction of the zenith. An atmospheric dispersion corrector (ADC) is required to correct this effect if AO performance is not to be compromised. The most common type of ADC uses rotating prisms to cancel the atmospheric dispersion [15]. There is another approach using a pair of prisms with variable separation [16]. Such a system, suitable for use on an ELT has recently been described [17]. Another effect of angular refraction, which is of relevance to AO, is that the center positions of the image at visible and infrared wavelengths coincide only at the zenith. If wavefront sensing is carried out in the visible and the corrected science image is in the infrared, there will be a zenith-dependent offset between the image centers on the wavefront sensor and the science camera. This would be corrected by using a single ADC for both channels, but in practice it is very difficult to find glasses that will work well over such a broad spectral range. This effect has been considered by Roe [18], and he refers to it as “atmospheric differential refraction.” It can be taken into account by postprocessing (if the exposure times are short enough) or by including zenith-dependent offsets in the AO control software or hardware.

An atmospheric dispersion corrector is required to maintain diffraction-limited imaging in the presence of atmospheric refraction. The first concern is that the dispersion correcting device will be controlled according to a formula relating atmospheric dispersion to atmospheric pressure, temperature, and relative humidity, and different formulas are available with those of Owens [19] and Ciddor [20] being the most widely used, although older, less accurate formulas are also sometimes used. The Ciddor equation has been adopted by the International Association of Geodesy (IAG) as the standard equation for calculating the index of refraction. The formula of Ciddor agrees with measurements by Birch and Downs [21] to one part in $10^5$, which is within the experimental uncertainty. Possibly of more importance is the effect of uncertainty in the input parameters. The refractivity of air depends on the temperature, pressure, relative humidity, and concentration of carbon dioxide. Using the Ciddor equations, an error of $5 \times 10^{-8}$ corresponds to an error of either 0.05°, 25 Pa, or 5% relative humidity. A similar error would be incurred by ignoring the contribution of carbon dioxide. What effect would these errors have on diffraction-limited imaging on an ELT? Hardy [22] presents an approximate expression for the effect of a random tilt error. If the diffraction-limited core is approximated to a Gaussian with standard deviation $\sigma_0 = 0.44\lambda/D$, then the volume of a core of height $A_0$ is equal to $2\pi A_0 \sigma_0^2$. A tilt error with standard
deviation $\sigma$, will broaden the core, increasing the variance to $\sigma^2 + \sigma^2$. The Strehl ratio, which is proportional to the peak intensity, is therefore reduced by a factor $\sigma^2 / (\sigma^2 + \sigma^2)$. The diffraction-limited full width at half-maximum of a 42 m telescope is 6 mas at a wavelength of 1 $\mu$m. A tilt error of 1 mas rms would reduce the Strehl ratio by a factor of 0.82, which would be a dramatic effect. The effect of error due to an inaccurate dispersion correction would actually be larger, since it would be nonrandom, and it would be accumulated during an exposure as the observation zenith distance is changing.

Assuming a spherically symmetric atmosphere, to a good approximation, an image is displaced by an amount given by the following formula [23], sometimes referred to as the Laplace equation:

$$\delta \theta = \kappa \alpha(1 - \beta) \tan(\xi_0) - \kappa \alpha(\beta - \alpha/2) \tan^2(\xi_0)$$  (4)

where $\xi_0$ is the apparent zenith distance at the ground, the parameter $\kappa = 1$ for a spherical Earth, $\beta$ is the ratio of atmospheric scale height to the geocentric distance of the observatory, and $\alpha$ is the refraction ($\alpha = n - 1$). This formula implies that an error of $5 \times 10^{-8}$ in refractive index would lead to an error of 1 mas over a zenith distance of approximately 5° (or less at high zenith distance), which is a moderate change in zenith distance during a long exposure (or series of exposures, which will be coadded).

The approximations in the above formula allow the atmospheric refraction to be calculated using the refractive index local to the observatory. Alternatively, tables are available that allow the refraction to be calculated based on empirical measurements and atmospheric models. The most widely used example is the Pulkovo tables [24]. Stone [23] compared values using the above formula and the Pulkovo tables and found differences of $< 5$ mas for zenith distances up to 60°.

More accurate results would be obtained by performing a ray tracing calculation, which would require knowing the atmospheric parameters along the line of sight. This was examined by van der Werf [25] who compared atmospheric refraction as computed by ray tracing the modified U.S. 1976 atmosphere with tables based on atmospheric models. Although the data are presented with only two decimal places, it appears from van der Werf’s comparison that there is a significant (> 20 mas) difference between ray tracing and the Pulkovo tables for zenith distances above 35°.

It therefore appears that atmospheric refraction prediction at the level of a few milliarcseconds would indeed require knowledge of atmospheric parameters along the line of sight and ray tracing. When this is possible, the correction should be calculated for the effective wavelength of the observation, which depends on the effective temperature of the star being observed, the spectral transmission of the atmosphere, telescope and instrument, and the wavelength dependence of the detector quantum efficiency. Malyuto and Meinel [26] calculate that an accuracy of 80–140 K in effective stellar temperature is necessary to achieve an accuracy of 20 mas, and they state that this is what is possible with current spectral and photometric classification systems. The effect of error in effective temperature is greatest for cool stars, such as M or K dwarfs. For stars in the galactic plane, the effects of interstellar reddening should also be taken into account. Ideally, a real spectrum of the object would be used, but this is rarely available.

A solution could be to use a dispersion sensor and control the ADC in closed loop. This sensor would pick off a star outside the AO field; the star could be faint, as the correction would be relatively slow. The apparent position of the star would be monitored at two different wavelengths; these should be as widely spaced as possible, to maximize the differential signal, while providing sufficient signal. Two detectors and a dichroic could be used, or a single detector with a Bayer-type filter, i.e., adjacent pixels made sensitive to different wavelengths. Further studies would be required to determine the requirements for such a system, including the field required for good sky coverage. It should be noted that a fast tip–tilt correction of similar accuracy will also be required as part of the AO operation.

B. Chromatic Path Length Error
The optical path length error introduced by turbulence will be slightly different at different wavelengths due to dispersion. A deformable mirror will apply a correction, which will be perfect at a single wavelength. There will therefore be an error at the other wavelengths in the correction band. This error may be estimated according to the following reasoning [27]. If a deformable mirror applies perfect correction at $\lambda_0$, then the error in phase at wavelength $\lambda$ is given by

$$\delta \phi(\lambda) = \epsilon(\lambda, \lambda_0) \phi(\lambda_0),$$  (5)

where

$$\epsilon(\lambda, \lambda_0) = \frac{\lambda_0 n_s(\lambda) - n_s(\lambda_0)}{\lambda_0 n_s(\lambda_0) - 1},$$  (6)

and $n_s$ is the refractive index of air at standard conditions of pressure and temperature. If the rms uncorrected wavefront phase error is $\sigma_p$, then the residual chromatic error variance at wavelength $\lambda$ will be given by

$$\sigma_{ch}^2(\lambda) = \epsilon^2(\lambda, \lambda_0) \sigma_p^2.$$  (7)

Consider as an example the case of a 42 m telescope operating when the seeing at 0.5 $\mu$m corresponds to a Fried parameter of 0.15 m at 0.5 $\mu$m.
The piston-removed variance (in square radians) is given by Noll's formula [28]

\[ \sigma_u^2 = 1.03 \left( \frac{D}{r_0} \right)^{5/3} . \]  

(8)

If the correction is perfect at \( \lambda_0 = 0.7 \mu m \), then the Strehl ratio (as calculated using the extended Maréchal approximation) falls below 0.9 for correction wavelengths shorter than 0.63 \( \mu m \) or longer than 0.84 \( \mu m \). However, the uncorrected wavefront variance is grossly overestimated using Noll's formula as it assumes an infinite outer scale, \( L_0 \). Owner-Petersen and Goncharov [27] calculated a factor, \( g(D/L_0) \), which should be multiplied by Noll's variance. This factor drastically reduces the uncorrected variance if the telescope aperture diameter is larger than the outer scale. Measurements of the outer scale range from meters [29] to kilometers [30], although it seems that more typical values are in the range of a few tens of meters. Ziad et al. [31] used three techniques to measure the outer scale at Palomar Observatory and generally found values in the range of 5–20 m. Figure 3 shows the corresponding Strehl ratio as a function of correction wavelength for \( \lambda_1 = 0.589 \mu m \), \( D = 42 \text{ m} \), and \( D/L_0 = 0 \), 1, 2. The effect could be important whenever the outer scale is large. It has been pointed out [3] that the chromatic path length error can be corrected at the central correction wavelength if the dispersion is known. This will be difficult in practice, as pointed out in Section 2.

C. Chromatic Anisoplanatism

Rays of different colors that are coincident at the top of the atmosphere will separate and travel through different paths in the atmosphere due to atmospheric dispersion. The fact that the rays travel through different paths in the turbulence leads to an isoplanatic error. This was studied in a number of papers by Wallner. He first derived an expression for the phase variance assuming a large telescope and the guide star at infinity [6]. In a subsequent publication [7], he allowed for a finite sized telescope aperture and the guide star to be at a finite distance. Sasiela [32] studied the effect in a paper on isoplanatic errors and refers to it as chromatic anisoplanatism. He derived an expression for wavefront variance, which can be shown to reduce to Wallner's. He also presents an expression allowing the Strehl ratio to be calculated more accurately than using the Maréchal approximation in the case of anisoplanatic errors. Finally, Nakajima [33] has recently published an article dedicated to this effect, which he refers to as “chromatic shear.” His expression is practically identical to those previously derived, and he applies the theory to a Mauna Kea model turbulence profile. Neither Nakajima nor Sasiela refer to Wallner's papers, nor does Nakajima refer to Sasiela, and so we have carefully compared their analyses. It can be seen that their approaches are basically equivalent. The first step is to derive an expression for the displacement of beams of different wavelengths as a function of altitude. This depends on the refractive index as a function of altitude, and if the atmospheric composition is assumed constant, then the refractive index only depends on the air density. This is referred to as the Gladstone–Dale law, and Sasiela uses it explicitly in his expression for the chromatic displacement

\[ d_c(z) = \frac{\sin(\xi)\Delta \alpha_0}{\cos^2(\xi)} \int_0^z dz' \alpha(z') , \]

(9)

where \( \Delta \alpha_0 \) is the difference in refractive index between wavelengths \( \lambda_1 \) and \( \lambda_2 \), \( \alpha \) is the air density normalized to its value at sea level, and \( \xi \) is the zenith angle. Nakajima takes an equivalent approach, assuming that refractive index scales with pressure divided by absolute temperature. Wallner points out that, assuming a flat Earth, the integral in the above expression is equal to the atmospheric pressure at \( h \) divided by the acceleration of gravity. The authors used different (but very similar) atmospheric models. We will use the model employed by Nakajima, which is available online [34]. Figure 4 shows the chromatic displacement...
displacement between rays at 0.589 and 2.2 μm as a function of altitude using the Sasiela formula for three different zenith angles. It can be seen that the displacement is quite small compared to typical values of the Fried parameter over this wavelength range. It is therefore expected that the wavefront error will be small but it may be important in the case of extreme AO.

The next step in the analysis is to determine the structure function for the phase difference between the waves at different wavelengths. The Kolmogorov structure function for the phase difference between the waves at different wavelengths. The Kolmogorov structure function for the phase difference between rays at 0.589 and 2 μm is the second distance moment defined according to

\[ d_m = 2.91k_0^2 \int_0^\infty dz C_n^2(z) d_n(z). \]

We now apply the formulas using the seven-layer ORM turbulence model. Figure 5 shows the Strehl ratio as a function of wavelength when the wavefront sensing is at 0.589 μm. The results of the analysis of Nakajima and Wallner are practically identical while Sasiela gives a slightly higher value for the Strehl ratio since it allows a more accurate calculation than using the Maréchal formula.

4. Partial Correction of Chromatic anisoplanatism

Wallner [7] pointed out that, if all the turbulence were situated at a high altitude, then the rays at different wavelengths experience the same phase error before separating due to dispersion. The wavefront correction at different wavelengths could be perfect if they are brought together before the deformable mirror. If, on the other hand, the turbulence were all located at or near the pupil, then there would be no need to introduce a chromatic shear. In reality, the turbulence is distributed through the atmosphere, and there will be an optimal shear that depends on the distribution of turbulence in altitude. We have studied this effect for both the ORM seven-layer model and a six-layer model of the turbulence above Mauna Kea [35], scaled to give the same seeing as the ORM profile. Figure 6 shows the Strehl ratio calculated using the Maréchal formula for \( \lambda_1 = 0.589 \mu m \) and \( \lambda_2 = 2.2 \mu m \) as a function of the fractional residual shift, i.e., the origin corresponds to
correcting the full chromatic shift (which can be seen in Fig. 4) while unity corresponds to not shifting. It can be seen that in both cases there is a small gain in performance to be obtained by partially shearing the wavefronts at these wavelengths. The optimal shear is approximately 20% of the full atmospheric shear in the case of the ORM and ~50% in the case of Mauna Kea. The larger value for the Mauna Kea profile is a result of the higher contribution from high altitude turbulence. It is remarkable that an overcorrection can lead to a degradation of performance. This could happen inadvertently due to optics before the deformable mirror, an example of which could be an ADC. Figure 7 demonstrates the chromatic pupil shift due to a linear ADC such as we recently proposed for use on ELTs [17]. In this case the exit pupil, \( P' \), is situated at some finite distance \( S_F \) from the focal plane \( I' \). The chromatic shift referenced to the entrance pupil, \( C' \), is related to the shift relative to the exit pupil, \( C \), by

\[
\frac{C'}{C} = \frac{f'}{S_F},
\]

where \( f' \) is the effective focal length of the telescope. From Fig. 7 it can be seen that \( C = AS_F \), where \( A \) is the angular dispersion between the two wavelengths. Finally, we have \( A = A'/M \), where \( M \) is the pupil magnification and \( A' \) is the angular dispersion on the sky. For a two-mirror telescope a typical value is \( M = 0.1 \). For any ADC design, the chromatic shift should be calculated and the effect determined using Fig. 6. Indeed, the ADC design can be adjusted to give a chromatic shift near the optimal.

Alternatively, it may be desirable to ensure that the chromatic shift introduced by the ADC is zero for all wavelengths. This could facilitate the science instrument placed after the ADC. It could also be of benefit in a multiconjugate AO system. In this case, the deformable mirrors are conjugate to the main layers of turbulence, and the chromatic shift will be considerably reduced in comparison to a single-conjugate system. The above analysis shows that zero pupil shift can be obtained if the exit pupil is imaged to infinity, i.e., if the ADC is in a telecentric path. Figure 8 shows a possible configuration. It can be seen that the chief rays at different wavelengths enter the center of the entrance pupil at different angles due to the atmospheric dispersion, which in turn gives rise to an elongated object in image plane \( I \). The first relay lens forms an image of the telescope pupil, \( P' \), while the second relay lens reimages the dispersed star image through the linear ADC, so that images at two wavelengths are brought together at the final image plane \( I' \). The image at the third wavelength is left unshifted because the ADC is not able to correct atmospheric dispersion over a broad wavelength range. There is always some residual difference between the atmospheric dispersion and that of the glass used for wedges. Nevertheless, all three wavelengths are brought together at the final pupil image \( P'' \).

We note that, in principle, the chromatic shift between the wavefront sensing wavelength and the correction wavelengths could be applied in the AO control system. However, a zenith-dependent shift of the wavefront correction adds to the complexity of the control and could adversely affect the stability of the closed loop.

![Fig. 7. (Color online) Pupil shift introduced by linear ADC in a nontelecentric path.](image1)

![Fig. 8. (Color online) Schematic optical layout of the telescope with a linear ADC working in the telecentric path. The solid curves represent the chief rays at three different wavelengths; the dotted lines are the corresponding marginal rays.](image2)
5. Effect of Chromatic Errors on the Point Spread Function

So far in this paper the effect of chromatic errors on residual wavefront variance and the Strehl ratio have been considered. A reduction in the Strehl ratio will lead to light being scattered out of the diffraction-limited image core. However, it is of interest to know how the scattered light is distributed around the core as this will impact such tasks as faint companion or exoplanet detection. We use a simple point spread function (PSF) model based on an analytical description of the residual phase errors [36]. This approach has been employed previously to estimate the chromatic effects on the corrected PSF [3,37]. For very large telescopes, the form of the PSF can be approximated by

$$PSF(f) = S \cdot PSF_D(f) + \frac{4}{\pi D^2} W(f),$$  \hspace{1cm} (17)

where S is the Strehl ratio, PSF_D is the diffraction-limited PSF, W(f) is the residual phase spectrum as a function of spatial frequency f, and D is the telescope aperture diameter. In order to evaluate this expression at an angular offset, a radians from the center, the substitution $f = \alpha/\lambda$ is made. The second term in the above expression can be considered the chromatic background. It will correspond to random speckles in short-exposure images, averaging to a smooth halo in long exposure images. In the case of the chromatic path length error, due to wavefront sensing at $\lambda_1$, while correction is required at $\lambda_2$, the chromatic background at $\lambda_2$ is given by

$$PSF_{bg}(f) = \epsilon^2(\lambda_1, \lambda_2) W_{atm}(f, \lambda_1),$$  \hspace{1cm} (18)

where $\epsilon(\lambda_1, \lambda_2)$ is the dispersive error factor introduced in Subsection 3.B. $W_{atm}(f, \lambda)$ is the uncorrected atmospheric phase spectrum, an expression for which is

$$W_{atm}(f, \lambda) = \frac{0.0229}{\lambda_0^{5/3}(\lambda)} \left( f^2 + L_0^{-2} \right)^{-11/6},$$  \hspace{1cm} (19)

where $L_0$ is the outer scale of turbulence. Figure 9 shows the background term for $D = 42$ m, $\lambda_1 = 0.8 \mu$m, and $\lambda_2 = 0.9 \mu$m in the case of having an infinite outer scale and an outer scale of $20$ m. We have already seen that a small outer scale greatly reduces the chromatic path length error. From Fig. 9 we can see that the reduction is most significant in the inner part of the PSF, thereby facilitating the search for faint companions near the diffraction limit ($0.005$ arc sec in this case).

A similar analysis can be carried out in the case of chromatic anisoplanatism. In this case we use an expression for the residual phase spectrum due to anisoplanatism in layered turbulence [36]
longer integration times to reach a given signal-to-noise ratio, and the chromatic component may limit the zenith distance and wavelength range over which planet detection is feasible.

6. Conclusions
We have reviewed the chromatic effects of the atmosphere in order to determine their effect on the next generation of very high-order AO, in particular on future ELTs. We confirmed that the chromatic dependence of scintillation should have a small effect, of the order of 1% at a zenith angle of 45°. The first refractive effect, which is classical angular refraction, will require precise correction using atmospheric dispersion compensators. If very high Strehl ratios are to be maintained over significant exposure times, then closed-loop dispersion correction will be necessary. This will require the development of dispersion sensors and one possibility is to sense the position of an image at two wavelengths. The next effect, chromatic path length error, will be a concern whenever the outer scale is large in relation to the telescope aperture. The fraction of time during which this will be the case is poorly known and should be a subject for ELT site selection campaigns. The final effect considered has been chromatic anisoplanatism. We have confirmed that three published analyses of this effect are in excellent agreement. The effect may be significant for high-order correction, and it may be reduced by use of a chromatic wavefront shearer. We demonstrate, however, that overcorrection can lead to significant degradation in performance. This could be brought about in optics before the deformable mirror, for example, in the ADC. Finally, we have examined the effect of these errors on the uncorrected halo levels in the corrected PSF. The backgrounds are significant when compared to the contrast ratio required for the detection of exoplanets (10^{-6}–10^{-9}). In the case of chromatic path length error, the background near the diffraction-limit depends on the value of the outer scale. The background due to chromatic anisoplanatism depends strongly on zenith distance.

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